Cube Roots of Negative One "the tooth"

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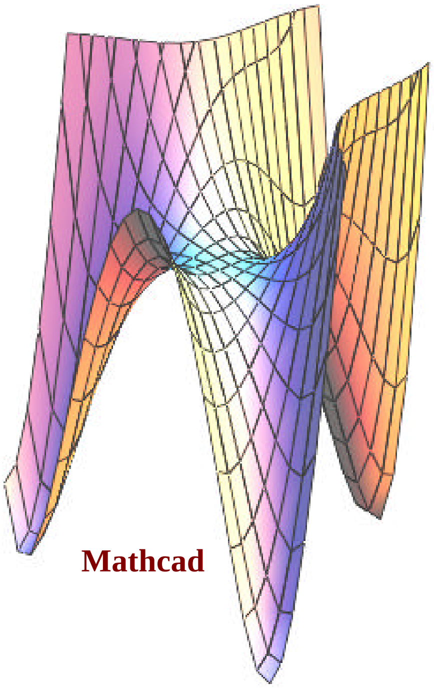

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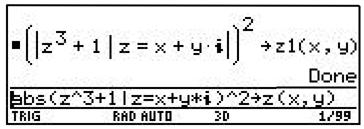
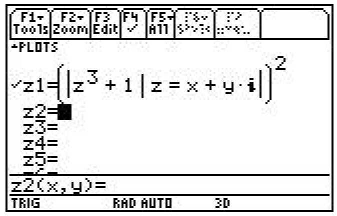
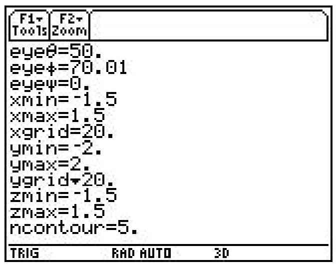


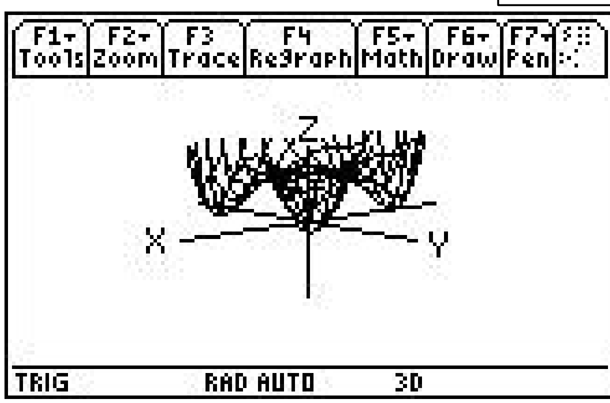




The function $z(x,y) = |f(z)|^2$ where $f(z) = z^3 + 1$ and $z = x + yi$, produces the graph of a polynomial function $z(x,y)$ whose zeros "point" to the cube roots of negative one. The zeros are the only function values that "produce" points that lie in the *xy*-plane (complex plane, if you will…). The zeros are the points in the *xy*-plane at the tips of the legs...note that $z(0,0) = 1$ (about the "middle" of the "3-legged-saddle" is at (0,0,1)).
It turns out that $f(z)$ is an imaginary-valued function except at the zeros and domain values along the *x*-axis (its domain is complex), and $z(x,y)$ is a real-valued polynomial (with complex domain) that is also modulus $f(z)$ squared.
$z(x,y) = |f(z)|^2 = |f(x + yi)|^2$.
The squaring could be left out (as below left/right) except that the hand calculations might be a little harder. This same process can be used to "view" complex roots of other functions (whose domains are real) **see below left/right.

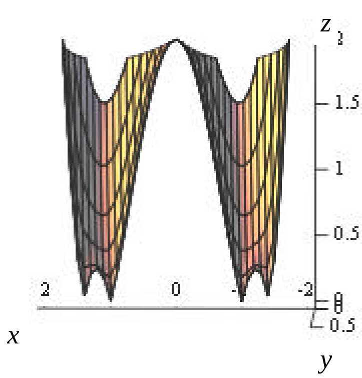
Professor Ira Rosenthal who is currently Chairperson of the Math Department at PBCC Eissey Campus brought this process to my attention. It originated at a math conference that she attended. The original presentation was given by Dr. Bert Waits (Professor Emeritus of Ohio University) - to whom the original idea belongs.

I, only, am responsible for errors in this document…if any ☺

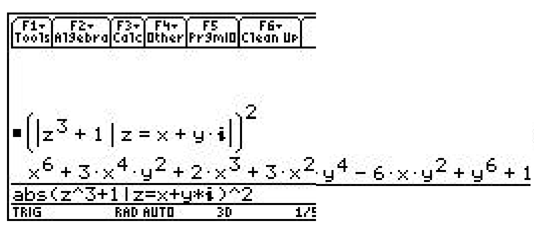


$z(x,y) = 0$ "produces" points in the *xy*-plane at (-1,0), (1/2,-sqrt(2)), and (1/2,sqrt(2)) so that the complex solutions of the equation $f(z) = z^3 + 1$ are $-1 + 0i$, ½-sqrt(2)*i*, and ½ + sqrt(2)*i*

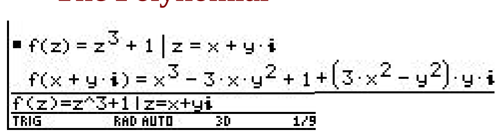
Treat the *xy*-plane as the complex plane
(Then *z* represents the length of *f*(*x* + *yi*), squared)



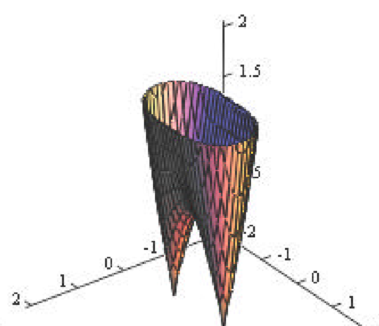
** $h(z) = z^4 - 3z^2 + 2$,) has complex roots 1, -1, sqrt(2), and -sqrt(2) that can be seen from the graph of $z(x,y)$ (above) when looking head-on at the *y*-axis (or *i*-axis) since when $z(x,y) = |(x + yi)^4 - 3(x + yi)^2 + 2|$ is graphed, $z(x,y) = 0$ "produces" points in the *xy*-plane at (1,0), (-1,0), (sqrt(2),0), and (-sqrt(2),0). (In this case, the roots are real.)
Note: This shows the solutions of $h(z) = z^4 - 3z^2 + 2$.



The Polynomial



The Function *f*(*z*)



** $h(z) = z^2 + z + 1$, has complex roots -½ + sqrt(3)/2 and -½- sqrt(3)/2, that can be seen from the graph of $z(x,y)$.
When $z(x,y) = |(x + yi)^2 + (x + yi) + 1|$ is graphed, $z(x,y) = 0$ "produces" points in the *xy*-plane at (-½,sqrt(3)/2) and (-½,-sqrt(3)/2). (In this case, the roots are imaginary.)
Note: This shows the solutions of $h(z) = z^2 + z + 1$.

May 1, 2001