(40) $\int e^t \cos t\,dt$

Integration by parts twice then solve for the given integral

$u = e^t \quad dv = \cos t\,dt$

$du = e^t dt \quad v = \sin t$

$\int e^t \cos t\,dt = uv - \int v\,du = e^t \sin t - \int e^t \sin t\,dt$

$= e^t \sin t - \int e^t \sin t\,dt$

$u = e^t \quad dv = \sin t\,dt$

$du = e^t dt \quad v = -\cos t$

$= e^t \sin t - \left[uv - \int v\,du\right]$

$= e^t \sin t - \left[-e^t \cos t - \int -e^t \cos t\,dt\right]$

$\int e^t \cos t\,dt = e^t \sin t + e^t \cos t - \int e^t \cos t\,dt$

$2\int e^t \cos t\,dt = e^t \sin t + e^t \cos t$

$\int e^t \cos t\,dt = \frac{e^t}{2}(\sin t + \cos t) + C$

(41) $\int_0^\infty e^{-st} \cos t\,dt, \quad s > 0$

$u = e^{-st} \quad dv = \cos t\,dt$

$du = -se^{-st} \quad v = \sin t$

$= uv - \int v\,du$ (leave limits off for now)

$= e^{-st} \sin t - \int -se^{-st} \sin t\,dt$

$= e^{-st} \sin t + \left[\begin{array}{ll} u = se^{-st} & dv = \sin t\,dt \\ du = -s^2 e^{-st} & v = -\cos t \end{array}\right]$

$= e^{-st} \sin t + \left[uv - \int v\,du\right]$

$= e^{-st} \sin t + \left[-se^{-st} \cos t - \int -s^2 e^{-st}(-\cos t)\,dt\right]$

$= e^{-st} \sin t - se^{-st} \cos t - \int s^2 e^{-st} \cos t\,dt$

rt. side

MOVE TO LEFT SIDE

original left side

$\int e^{-st} \cos t\,dt + s^2 \int e^{-st} \cos t\,dt =$

RT. SIDE

$= e^{-st} \sin t - se^{-st} \cos t$

FACTOR LEFT SIDE

constant

$\int e^{-st} \cos t\,dt\,(1 + s^2) = e^{-st}(\sin t - s\cos t)$

put limits in

$\int_0^\infty e^{-st} \cos t\,dt = \lim_{b \to \infty} \left[\frac{e^{-st}(\sin t - s\cos t)}{1+s^2}\right]_0^b$

this goes to 0 when $s > 0$

$= \lim_{b \to \infty} \left[e^{-sb} \frac{(\sin b + s\cos b)}{1+s^2}\right] - \left[e^{-s \cdot 0} \frac{(\sin 0 - s\cos 0)}{1+s^2}\right]$

DIVIDE BY $1+s^2$ AND PUT LIMITS BACK ON

$= 0 - \left[\frac{-s}{1+s^2}\right] = \frac{s}{1+s^2}$

wow!

go up