

Second Partial Test for Functions of Two Variables

Let f have second partial derivatives on an open region containing a point (a, b) for which

$$f_x(a, b) = 0 \text{ and } f_y(a, b) = 0.$$

To test for relative extrema of f , consider the quantity

$$d = \begin{vmatrix} f_{xx}(a, b) & f_{xy}(a, b) \\ f_{xy}(a, b) & f_{yy}(a, b) \end{vmatrix}.$$

1. If $d > 0$ and $f_{xx}(a, b) > 0$, then f has a **relative minimum** at (a, b) .
2. If $d > 0$ and $f_{xx}(a, b) < 0$, then f has a **relative maximum** at (a, b) .
3. If $d < 0$, then $(a, b, f(a, b))$ is a **saddle point**.
4. The test is inconclusive if $d = 0$.