

Basic Algebraic Exponent Properties $a^x a^y = a^{x+y}$ $a^x / a^y = a^{x-y}$ $(a^x)^y = a^{xy}$ $a^{-x} = 1/a^x$

Formulas for Compound Interest

For n compoundings per year: $A = P \left(1 + \frac{r}{n} \right)^{nt}$

For continuous compounding: $A = Pe^{rt}$ Present value: $A = Pe^{-rt}$ where $e = \lim_{x \rightarrow \infty} (1 + 1/x)^x \approx 2.71828182842\dots$

Definition of a Base e and Base 10 Logarithmic Function:

$y = \ln x$ or $y = \log x$ if and only if $x = e^y$ or $x = 10^y$

Note: These properties apply to bases other than e and 10 as long as the base is not negative, 1 or 0.

Logarithmic Function with base 10: $f(x) = \log x$

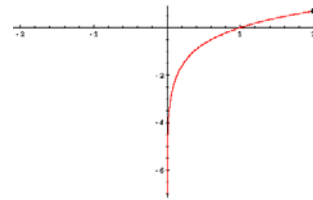
Exponential Function with base 10: $f(x) = 10^x$

Logarithmic Function with natural base e : $f(x) = \ln x$

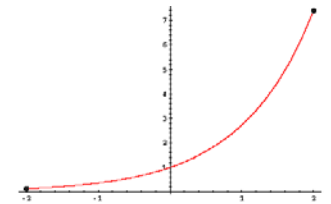
Exponential Function with natural base e : $f(x) = e^x$

Properties of Logarithms:

1. $\log 1 = 0$ (log of 1 is 0)
 $\ln 1 = 0$ (ln of 1 is 0)
2. $\log 10 = 1$ (log of the base is 1)
 $\ln e = 1$ (ln of the base is 1)
3. $\log 10^x = x$ (inverse property)
 $\ln e^x = x$ (inverse property)
4. $10^{\log x} = x$ (inverse property)
 $e^{\ln x} = x$ (inverse property)
5. If $\log x = \log y$ then $x = y$
6. If $e^x = e^y$ or $10^x = 10^y$ then $x = y$



$f(x) = \ln x$



$f(x) = e^x$

INVERSES

$f(x) = \log x$ and $f(x) = 10^x$ are inverses.

$f(x) = \ln x$ and $f(x) = e^x$ are inverses.

Change of Base: $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$ and $b^x = e^{kx}$ for $k = \ln b$

Properties of Logarithms:

1. $\log uv = \log u + \log v$ $\ln uv = \ln u + \ln v$ (product rule)
2. $\log \frac{u}{v} = \log u - \log v$ $\ln \frac{u}{v} = \ln u - \ln v$ (quotient rule)
3. $\log u^n = n \log u$ $\ln u^n = n \ln u$ (power rule)

Exponential Growth and Decay: $y' = ky \Leftrightarrow y = Ce^{kx}$.

Carbon Dating: $R = \frac{1}{10^{12}} e^{-t/8223}$

Sales Decay: $s(t) = s_0 e^{-\lambda t}$ **Gaussian Models:** $y = ae^{-(x-b)^2/c}$ **Normally Distributed:** $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$ (mean = 0)

Time Constant: $y = Ce^{-t/T}$ **Logistics Growth Model:** $y = \frac{M}{1 + Be^{-Mkt}}$, $y' = ky(M - y)$

Terminal Velocity M : $v(t) = M(1 - e^{-kt})$ **Carbon Dating/Exponential Decay:** $P(t) = P_0 e^{-\lambda t}$

Magnitude of Earthquakes: $R = \log \frac{I}{I_0}$ **Learning Curve M syllables:** $f(t) = M(1 - e^{-kt})$

Newton's Law of Cooling: $D = D_0 e^{kt}$ where D_0 is the initial difference in temperature and D is the difference at time t .

DERIVATIVES – Log and Exponential

$\frac{d(e^u)}{dx} = e^u u'$ ex) $\frac{d(e^{5x^2+x})}{dx} = e^{5x^2+x} (10x + 1)$

$\frac{d(\ln x)}{dx} = \frac{1}{x} = x^{-1}$

$\frac{d(\ln u)}{dx} = \frac{1}{u} u' = \frac{u'}{u}$ ex) $\frac{d(\ln(5x^2 + x))}{dx} = \frac{10x + 1}{5x^2 + x}$

Elasticity of Demand:

$E(p) = \frac{-pf'(p)}{f(p)}$

Growth and Decay

$y' = ky \Leftrightarrow y = Ce^{ky}$

(The change in y is proportional to the amount present at any given time.)

ANTIDERIVATIVES – Indefinite Integral

$\int e^{kx} dx = \frac{1}{k} e^{kx} + C$

$\int \frac{1}{x} dx = \ln|x| + C = \begin{cases} \ln x + C_1 & \text{for } x > 0 \\ \ln -x + C_2 & \text{for } x < 0 \end{cases}$

See section 5.4 for more.
Calculus...Goldstein...11th Ed.