Use Direct Substitution to find $\lim_{x \to c} f(x)$ when *c* is in the Domain of these Functions and *c* is real

[i.e. $\lim_{x \to c} f(x) = f(c)$]

a) *p*(*x*) and *q*(*x*) are polynomials and *p*(*c*) ¹ 0

1) $f(x) = p(x)$

2) $f(x) = p(x)/q(x)$

3) $f(x) = \sqrt[n]{x}$, when *n* is odd

4) $f(x) = \sqrt[n]{x}$, when *n* is even and $c > 0$

5) $f(x) = \sin(x)$, $f(x) = \cos(x)$, $f(x) = \tan(x)$, $f(x) = \csc(x)$, $f(x) = \sec(x)$, $f(x) = \cot(x)$,

6) When $f(x) = g(x)$ for all *x* not equal to *c*, *g* is a reduced form of *f*, and *g* is defined at *c*. Then $\lim_{x \to c} f(x) = g(c)$

b) Special Trig Limits

1) $\lim_{x \to 0} \frac{\sin x}{x} = 1, \quad \lim_{x \to 0} \frac{\sin ax}{x} = a$

2) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0, \quad \lim_{x \to 0} \frac{1 - \cos ax}{x} = 0$

3) $\lim_{x \to 0} \frac{\tan x}{x} = 1, \quad \lim_{x \to 0} \frac{\tan ax}{x} = a$ [follows from 1) and direct substitution]

Strategies

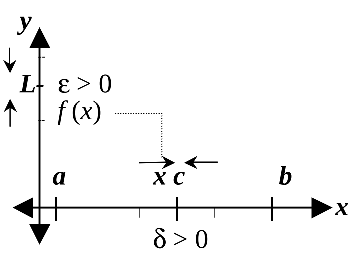
1) Use direct substitution on types a1) through a6)

2) Try using b), reducing, rationalizing, using other trig identities or otherwise find a function *g* that agrees with *f* except maybe at *c*

Definition of Limit

For

1) $f(x)$ defined on an open interval, $a < x < b$
2) $f(x)$ is defined except possibly at *c*
3) *c* is also in the interval, $a < c < b$



$$\lim_{x \to c} f(x) = L$$

means that for any number ε > 0, no matter how small,
there exists a number δ > 0 such that as *x* gets within δ of *c*,
that is, $0 < |x - c| < \delta$,
then $f(x)$ gets within ε of *L*,
that is, $| f(x) - L | < \varepsilon$.