limits.doc 5/16/02 6:56 AM

Limits - Brief Definitions, Etc.

Definition: If $f(x)$ becomes arbitrarily close to L as x approaches c from either side, then $\lim_{x\to c} f(x) = L$.

Properties

1) The limit of a constant function is the constant

$f(x) = b;\ \lim_{x\to c} b = b$

2) The limit of the identity function as x approaches c is c

$f(x) = x;\ \lim_{x\to c} x = c$

3) The limit of x to the n as x approaches c is x to the c

$f(x) = x^n;\ \lim_{x\to c} x^n = c^n$

4) The limit of the nth root of x as x approaches c is the nth root of c

$f(x) = \sqrt[n]{x};\ \lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$

Operations

1) The limit of a constant times a function is the constant times the limit of the function

$bf(x);\ \lim_{x\to c} bf(x) = b\lim_{x\to c} f(x)$

2) The limit of the sum of two functions is the sum of the limits of the functions

$f(x) \pm g(x);\ \lim_{x\to c}\left[f(x) \pm g(x)\right] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

3) The limit of the product of two functions is the product of the limits of the functions

$f(x)g(x);\ \lim_{x\to c}\left[f(x)g(x)\right] = \lim_{x\to c} f(x) \lim_{x\to c} g(x)$

4) The limit of the quotient of two functions is the quotient of the limits of the functions (if the divisor is not 0)

$f(x)/g(x);\ \lim_{x\to c}\left[f(x)/g(x)\right] = \lim_{x\to c} f(x) / \lim_{x\to c} g(x),\ g(x) \neq 0$

5) The limit of a function to a power is the limit of the function raised to the power

$\left[f(x)\right]^n;\ \lim_{x\to c}\left[f(x)\right]^n = \left[\lim_{x\to c} f(x)\right]^n$

6) The limit of the root of a function is the root of the limit of the function

$\sqrt[n]{f(x)};\ \lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$

Polynomial Function (a_n is not zero)

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Limit of a Polynomial Function

If f is a polynomial function and c is any real number, then

$\lim_{x\to c} f(x) = f(c)$.

The Replacement Theorem

Let c be a real number and $f(x) = g(x)$ for all x not equal to c. If the limit of $g(x)$ exists as x approaches c, then the limit of $f(x)$ also exists and $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$. In other words, if you can (legally) alter a function by reducing it or some other process, the limit of the altered function will be the same as that of the original function.

Existence of a Limit

A limit exists if and only if the left and right limits are equal.