

## Implicit Differentiation Trick – for applied calculus students

(Note: Not really a trick but requires more advanced calculus to understand why it works)

This method greatly simplifies implicit differentiation.

You need to know how to take partial derivatives,  $F_x$  and  $F_y$ ...very simple...treat everything as a constant except the variable of interest.

If  $F(x, y) = x^2 + y^2 - 4$ , then

$$F_x(x, y) = 2x \text{ and } F_y(x, y) = 2y.$$

For  $F_x(x, y)$ , treat everything in  $x^2 + y^2 - 4$  except  $x$  as a constant.

For  $F_y(x, y)$ , treat everything in  $x^2 + y^2 - 4$  except  $y$  as a constant.

$$\text{Then } \frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)} = -\frac{2x}{2y} = -\frac{x}{y}.$$

Example: Find  $\frac{dy}{dx}$  given  $y^3 + y^2 - 5y = x^2 - 4$ .

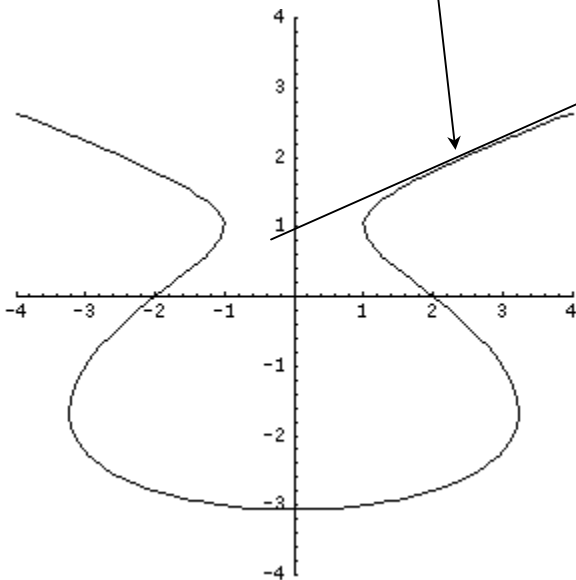
$$\text{Set equal to 0. } 0 = y^3 + y^2 - 5y - x^2 + 4.$$

$$\text{Define } F \text{ as } F(x, y) = y^3 + y^2 - 5y - x^2 + 4.$$

$$\text{Then you have } F_x(x, y) = -2x \text{ and } F_y(x, y) = 3y^2 + 2y - 5$$

$$\text{and it follows that } \frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)} = -\frac{-2x}{3y^2 + 2y - 5} = \frac{2x}{3y^2 + 2y - 5}.$$

To find the slope at a point  $(x, y) = (\sqrt{6}, 2)$ , put  $x = \sqrt{6}$  and  $y = 2$  in  $m = \frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5} = \frac{2 \cdot \sqrt{6}}{3 \cdot 2^2 + 2 \cdot 2 - 5} = .445 = m$ .



Graphed at:

<http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=graphs&s2=equations&s3=basic>