

# Applications of the Derivative (Derivative Uses with Steps)

(Assembled from known facts by Ron Mower Update/Print 9/9/06 12:36 PM) derivative\_uses\_steps.doc

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## Increasing/Decreasing Graph and Critical Numbers

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### Increasing/Decreasing

- At a point (at which  $f$  is defined)
  - If  $f'(a) > 0$ , then  $f$  is increasing at  $x = a$ .
  - If  $f'(a) < 0$ , then  $f$  is decreasing at  $x = a$ .
- On an interval (on which  $f$  is defined)
  - If  $f'(x) > 0$  for all  $x$  in an interval, then  $f$  is increasing on the interval.
  - If  $f'(x) < 0$  for all  $x$  in an interval, then  $f$  is decreasing on the interval.

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If function  $f$  is continuous at  $x = a$ ,  
it is not necessarily differentiable at  $x = a$ .

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### Critical Number

Suppose  $f$  is defined at some real number  $c$ ; that is,  $f(c)$  exists. If  $f'(c) = 0$  or  $f'(c)$  is undefined, then  $c$  is called a **critical number** of function  $f$ .

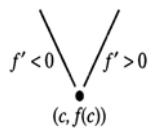
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## Relative Extrema and Curve Sketching

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### First Derivative Test

- Let  $c$  be a critical number of  $f$  and let  $f$  be continuous on an interval containing  $c$ . Then  $(c, f(c))$  is a *relative maximum point* provided that  $f'(x) > 0$  in an interval to the left of  $c$  and  $f'(x) < 0$  in an interval to the right of  $c$ .
- Let  $c$  be a critical number of  $f$  and let  $f$  be continuous on an interval containing  $c$ . Then  $(c, f(c))$  is a *relative minimum point* provided that  $f'(x) < 0$  in an interval to the left of  $c$  and  $f'(x) > 0$  in an interval to the right of  $c$ .



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## Concavity, the Second Derivative Test and Curve Sketching

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### Test for Concavity

The graph of  $f$  is

- Concave down where  $f''(x) < 0$ .
  - Concave up where  $f''(x) > 0$ .
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### Point of Inflection

Any point at which the graph of a continuous function changes concavity is called a **point of inflection**.

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### Second Derivative Test

Assume  $c$  is a critical number of  $f$ .

- If  $f''(c) < 0$  then  $(c, f(c))$  is a *relative maximum point*.
- If  $f''(c) > 0$  then  $(c, f(c))$  is a *relative minimum point*.

*Note:* If  $f''(c) = 0$  or  $f''(c)$  is undefined, you can draw no conclusion from the test. You must use the first derivative test instead.

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## Steps in Sketching the Graph of Function $f$

- Determine  $f'(x)$  and  $f''(x)$ .
  - Use  $f'(x)$  to find all critical numbers.
  - Apply the second derivative test to critical numbers in order to determine relative extrema. If the test fails, use the first derivative test.
  - Determine the  $y$ -coordinates of all relative maximum and relative minimum points.
  - Show the graph at relative extreme points as rounded (if  $f'(c) = 0$ ) or pointed (if  $f'(c)$  does not exist).
  - Determine where the graph is concave up and where it is concave down.
  - Determine points of inflection.
  - Determine one or more additional points as needed and complete the sketch.
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## Absolute Extrema

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### Finding Absolute Extrema

Every function  $f$  that is continuous on a closed interval  $[a, b]$  will have an absolute maximum and an absolute minimum on the interval. To find the absolute extrema,

- Find all critical numbers off that are in the interval  $(a, b)$ .
  - Evaluate  $f$  at each critical number and also compute  $f(a)$  and  $f(b)$ .
  - The largest value computed in step 2 is the *absolute maximum*. The smallest value that is computed in step 2 is the *absolute minimum*.
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## Applied Maximum/Minimum

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### Steps for Solving Maximum/Minimum Problems

- Read the problem carefully.
  - If possible, draw a picture.
  - Determine the constants and variables. Label parts of the drawing.
  - Establish an equation containing the variable to be maximized or minimized.
  - If the equation does not give the variable to be maximized or minimized as a function of *one* variable, then you must make a substitution in order to obtain a function of just one variable.
  - Now that the function is established, proceed to obtain critical numbers and so on-as was done in Examples 1 and 2.
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### As an alternative approach to finding maximum profit,

Profit is maximized when marginal revenue

$R'$  is equal to marginal cost  $C'$ , provided that  $R'' < C''$ .

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