

Critical Number Wednesday, October 13, 1999 4:18:27 PM - derivative\_uses.html

If the original function  $f$  is defined at  $c$ , then  $c$  is a critical number of  $f$  if  $f'(c) = 0$  or if  $f'$  is undefined at  $c$ . [Note that  $f'$  can be undefined at  $c$  and  $f(c)$  may be undefined also. In this case, the statement does *not* say that  $c$  is a critical number.]

### Increasing/Decreasing Intervals/Relative Max's/Relative Min's (Extrema)

#### Steps - First Derivative:

Find  $f'$ , set it equal to zero and find the critical numbers [zeros and points where  $f'$  is undefined (but  $f$  is defined)]

Using the critical numbers, partition the number line

Test an easy value of  $f'$  between the critical numbers

Record whether  $f'$  is + or - at the test point

#### Conclusions:

If  $f' > 0$  between critical numbers  $c$  and  $d$ ,  $f$  is *increasing* in the interval  $(c, d)$  [Note:  $c$  may be  $-\infty$  and  $d$  may be  $+\infty$ ]

If  $f' < 0$  between critical numbers  $c$  and  $d$ ,  $f$  is *decreasing* in the interval  $(c, d)$

For  $f$  defined at  $c$ ,  $f' > 0$  on the left side of  $c$  and  $f' < 0$  on the right side, there is a *relative maximum* at  $c$

For  $f$  defined at  $c$ ,  $f' < 0$  on the left side of  $c$  and  $f' > 0$  on the right side, there is a *relative minimum* at  $c$

### Absolute Max's/Absolute Min's on a Closed Interval

#### Steps for Finding Extrema on a Closed Interval $[a, b]$ where $f$ is continuous:

Evaluate  $f$  at each of the critical numbers in  $(a, b)$

Evaluate  $f$  at each endpoint,  $a$  and  $b$

#### Conclusions:

The least of these values is the *absolute minimum*

The greatest of these values is the *absolute maximum*

### Second Derivative Test for Concavity

#### Test:

For  $f$  differentiable on an open interval,  $I$ :

$f$  is *concave up* on  $I$  if  $f'' > 0$  on  $I$

$f$  is *concave down* on  $I$  if  $f'' < 0$  on  $I$

#### Steps - Second Derivative:

Find  $f''$ , set it equal to zero and find the c-critical numbers [zeros and points where  $f''$  is undefined (but  $f'$  is defined)]

Using the c-critical numbers, partition the number line

Test an easy value of  $f''$  between the critical numbers

Record whether  $f''$  is + or - at the test point

#### Conclusions:

If  $f'' > 0$  between c-critical numbers  $c$  and  $d$ ,  $f$  is *concave up* in the interval  $(c, d)$  [Note:  $c$  may be  $-\infty$  and  $d$  may be  $+\infty$ ]

If  $f'' < 0$  between c-critical numbers  $c$  and  $d$ ,  $f$  is *concave down*

Points where the concavity changes and a tangent line can be drawn are *points of inflection* and  $f''$  will either be 0 or undefined