chg_var_pol_cyl_sph.doc Sunday, April 15, 2001 10:53 PM

Integrals in Polar, Rectangular and Spherical Coordinates

Rectangular to polar

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(q)}^{g_2(q)} f(r\cos q, r\sin q)rdrdq$$

Rectangular to cylindrical

$$\iiint_Q f(x,y,z)dV = \int_{q_1}^{q_2}\int_{g_1(q)}^{g_2(q)}\int_{h_1(r\cos q, r\sin q)}^{h_2(r\cos q, r\sin q)} f(r\cos q, r\sin q, z)rdzdrdq$$

Rectangular to spherical

$$\iiint_Q f(x,y,z)dV = \int_{q_1}^{q_2}\int_{f_1}^{f_2}\int_{r_1}^{r_2} f(r\sin f\cos q, r\sin f\sin q, r\cos f)r^2\sin f drdfdq$$

(Reference) Cylindrical and Spherical Coordinates

Cylindrical Coordinates $P = (r,q,z)$; r,q as r,q before (polar), *z* is the directed distance between (r,q) and *P*.

To ***change coordinates between rectangular and cylindrical*** in equations, use:

Cylindrical to Rectangular: $x = r\cos q,\ y = r\sin q,\ z = z$

Rectangular to cylindrical: $r^2 = x^2 + y^2,\ \tan q = \dfrac{y}{x},\ z = z$

Spherical Coordinates $P = (r,q,f)$; r,q as r,q before, f is the angle between $\overrightarrow{OP}$ and the *z-axis*.

To ***change coordinates between rectangular and spherical*** in equations, use:

Spherical to Rectangular: $x = r\sin f\cos q,\ y = r\sin f\sin q,\ z = r\cos f,\ \sqrt{x^2+y^2} = r\sin f$

Rectangular to Spherical: $r^2 = x^2 + y^2 + z^2,\ \tan q = \dfrac{y}{x},\ f = \arccos\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)$

To ***change coordinates between cylindrical and spherical*** in equations, use:

Spherical to cylindrical ($r \geq 0$): $r^2 = r^2\sin^2 f,\ q = q,\ z = r\cos f$

Cylindrical to Spherical ($r \geq 0$): $r = \sqrt{r^2+z^2},\ q = q,\ f = \arccos\left(\dfrac{z}{\sqrt{r^2+z^2}}\right)$