Center of Mass and Moments of Inertia – Triple Integrals

$m = \iiint\limits_Q r(x,y,z)\,dV$ Mass of the solid

$M_{yz} = \iiint\limits_Q x\,r(x,y,z)\,dV$ First moment about *yz*-plane

$M_{xz} = \iiint\limits_Q y\,r(x,y,z)\,dV$ First moment about *xz*-plane

$M_{xy} = \iiint\limits_Q z\,r(x,y,z)\,dV$ First moment about *xy*-plane

$\bar{x} = \frac{M_{yz}}{m},\ \bar{y} = \frac{M_{xz}}{m},\ \bar{z} = \frac{M_{xy}}{m}$

$I_x = \iiint\limits_Q (y^2+z^2)\,r(x,y,z)\,dV$ Moment of inertia about *x*-axis

$I_y = \iiint\limits_Q (x^2+z^2)\,r(x,y,z)\,dV$ Moment of inertia about *y*-axis

$I_z = \iiint\limits_Q (x^2+y^2)\,r(x,y,z)\,dV$ Moment of inertia about *z*-axis

$I_x = I_{xz} + I_{xy},\ I_y = I_{yz} + I_{xy},\ I_z = I_{yz} + I_{xz},$

$I_{xy} = \iiint\limits_Q z^2\,r(x,y,z)\,dV$

$I_{xz} = \iiint\limits_Q y^2\,r(x,y,z)\,dV$

$I_{yz} = \iiint\limits_Q x^2\,r(x,y,z)\,dV$

Jacobians – Change of Variables – Double Integrals

If $x = g(u,v),\ y = h(u,v)$ $x = g(u,v),\ y = h(u,v)$ then the ***Jacobian*** of *x* and *y* with respect to *u* and *v* is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$, and $$\iint\limits_R f(x,y)\,dx\,dy = \iint\limits_S f(g(u,v),h(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv.$$