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<u>Components of Acceleration</u>

$$a_{\mathbf{T}} = \mathbf{a} \bullet \mathbf{T} = \frac{\mathbf{v} \bullet \mathbf{a}}{\|\mathbf{v}\|} = \frac{d^2 s}{dt^2}$$

$$a_{\mathbf{N}} = \mathbf{a} \bullet \mathbf{N} = \sqrt{\|\mathbf{a}\|^2 - a_{\mathbf{T}}^2} = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|} = K\left(\frac{ds}{dt}\right)^2$$

<u>Formulas for Curvature in the Plane</u>

$K = \left\|\frac{d\mathbf{T}}{ds}\right\| = \|\mathbf{T}'(s)\|$ ← **or space** a circle $K = \frac{1}{r}$

$K = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}}$ C given by $y = f(x)$

$K = \frac{|x'y'' - x''y'|}{\left[(x')^2 + (y')^2\right]^{3/2}}$ C given by $x = x(t)$, $y = y(t)$

<u>Formulas for Curvature in the Plane or Space</u>

$K = \|\mathbf{T}'(s)\| = \|\mathbf{r}''(s)\|$ s is arc length parameter

$K = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$ t is general parameter

$$K = \frac{\mathbf{a}(t) \bullet \mathbf{N}(t)}{\|\mathbf{v}(t)\|^2}$$

<u>Implicit</u> **<u>for *F(x,y,z)* = 0</u>** $\frac{\partial z}{\partial x} = -\frac{F_x(x,y,z)}{F_z(x,y,z)}$ and $\frac{\partial z}{\partial y} = -\frac{F_y(x,y,z)}{F_z(x,y,z)}$ (z implicit)

<u>Relative extrema</u> **<u>of *f*</u>**

$$d = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{vmatrix}.$$

1. If $d > 0$ and $f_{xx}(a,b) > 0$, then f has a **relative minimum** at (a, b).

2. If $d > 0$ and $f_{xx}(a,b) < 0$, then f has a **relative maximum** at (a, b).

3. If $d < 0$, then $(a, b, f(a, b))$ is a **saddle point**.

4. The test is inconclusive if $d = 0$.

<u>Arc Length of a smooth curve</u> $\mathbf{r}(t) = \overrightarrow{OP} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{j}$ is $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|dt$

<u>Arc Length Function along a smooth curve from a to b for</u> $a \le t \le b$. ***<u>s is the Arc Length Parameter</u>***

$$s(t) = \int_a^t \sqrt{[x'(u)]^2 + [y'(u)]^2 + [z'(u)]^2}\,du = \int_a^t \|\mathbf{r}'(u)\|du = \int_a^t \|\mathbf{v}(u)\|du$$