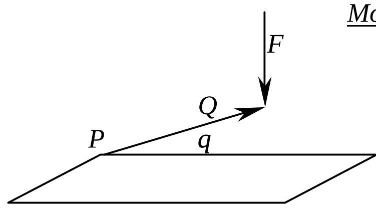
Angle between two nonzero vectors in standard position $0 \le q \le p$, $\cos q = \dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$; $\cos q \|\mathbf{u}\| \, \|\mathbf{v}\| = \mathbf{u} \bullet \mathbf{v}$; $\cos q \|\mathbf{u}\| = \dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{v}\|}$

***Projection of* u *onto* v:** $\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u} = \left(\dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}$ for $\mathbf{w}_1$ as above.

Force: The projection of **F** (force) onto **v** is given by $\text{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u}$ where ***v is the unit vector*** along a ramp. The ***magnitude of*** $\mathbf{w}_1$ ***is the force needed*** to keep the boat on the ramp.

Work: The work done by a constant force **F** as its point of application moves along the vector *PQ* is given by any of the following:

1) $W = \|\text{proj}_{PQ} \mathbf{F}\| \, \|\overrightarrow{PQ}\|$ projection form
2) $W = \cos q \|\mathbf{F}\| \|\overrightarrow{PQ}\|$ angle form
3) $W = \mathbf{F} \bullet \overrightarrow{PQ}$ dot product form



Moment of Force *F* about a point *P* is given by
$\mathbf{M} = \overrightarrow{PQ} \times \mathbf{F}$

Angle between two planes: $\cos q = \dfrac{|\mathbf{n}_1 \bullet \mathbf{n}_2|}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|}$.

Distance between a Plane and a Point Q (not in the plane): $D = \left\|\text{proj}_{\mathbf{n}} \overrightarrow{PQ}\right\| = \dfrac{\left|\overrightarrow{PQ} \bullet \mathbf{n}\right|}{\|\mathbf{n}\|}$ where *P* is a point in the plane and **n** is normal to the plane.

The ***distance between the point*** $Q(x_0, y_0, z_0)$ ***(not in the plane) and the plane*** given by $ax + by + cz + d = 0$ is

$D = \dfrac{|a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1) + d|}{\sqrt{a^2 + b^2 + c^2}}$ or $D = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$ where $P(x_1, y_1, z_1)$ is a point in the plane and $d = -(ax_1 + by_1 + cz_1)$.

Distance between a Point Q and a Line in Space: $D = \dfrac{\left\|\overrightarrow{PQ} \times \mathbf{u}\right\|}{\|\mathbf{u}\|}$ where **u** is the direction vector for the line and *P* is a point on the line.

Spherical Coordinates $P = (r, q, f)$; r, q as r, q before, f is the angle between $\overrightarrow{OP}$ and the *z-axis*.

To ***change coordinates between rectangular and spherical*** in equations, use:

Spherical to Rectangular: $x = r \sin f \cos q$, $y = r \sin f \sin q$, $z = r \cos f$, $\sqrt{x^2 + y^2} = r \sin f$

Rectangular to Spherical: $r^2 = x^2 + y^2 + z^2$, $\tan q = \dfrac{y}{x}$, $f = \arccos\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$

To ***change coordinates between cylindrical and spherical*** in equations, use:

Spherical to cylindrical ($r \ge 0$): $r^2 = r^2 \sin^2 f$, $q = q$, $z = r \cos f$

Cylindrical to Spherical ($r \ge 0$): $r = \sqrt{r^2 + z^2}$, $q = q$, $f = \arccos\left(\dfrac{z}{\sqrt{r^2 + z^2}} \right)$