ptch56_appcalc_05g10_sol.doc

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## ptch56 Solutions

**(1)**

$13.5 = 100e^{4k}$

$.135 \Leftarrow \frac{13.5}{100} = e^{4k}$

$\ln .135 = 4k$

$\frac{\ln .135}{4} = k$

- - - - - -

$y = Ce^{\frac{\ln .135}{4}t}$

$30 = 90e^{\frac{\ln .135}{4}t}$

$\frac{1}{3} = e^{\frac{\ln .135}{4}t}$

$\ln\frac{1}{3} = -\ln 3 = \frac{\ln .135}{4}t$

$t = \frac{-\ln 3}{\frac{\ln .135}{4}} = 2.195$ yrs

**(2)**

$3 = e^{24k}$

$\frac{\ln 3}{24} = k$

- - - - - -

$200 = 50e^{\frac{\ln 3}{24}t}$

$4 = e^{\frac{\ln 3}{24}t}$

$\frac{\ln 4}{\frac{\ln 3}{24}} = t$

$t = 30.3$ days

**(3)**

$\frac{dy}{dt} = \frac{1}{2}y$ (let $y = P(t)$)

formula

$y = Ce^{\frac{1}{2}t}$

- - -

$y(0) = 500 = Ce^{\frac{1}{2}(0)}$

$500 = C$

$y = 500e^{\frac{1}{2}t}$

(does not agree w/author's answer of $200e^{\frac{1}{2}t}$)

**(4)**

$A = Pe^{rt}$

$2 = e^{.10t}$

$\ln 2 = .10t$

$\frac{\ln 2}{.10} = t$

$t = 6.93$ years for both

**(5)**

$A = Pe^{rt}$ How long to \$1000

$1000 = 500e^{.05t}$

$2 = e^{.05t}$

$\frac{\ln 2}{.05} = t = 13.863$

after 13.863 years, the account is worth \$1000

Now, leave alone for 7 more yrs at 8%

$20 - 13.863 = 6.137$

$A = 1000e^{.08(6.137)}$

$A = 1633.88$ (does not agree w author's 2633.88)

**(6)**

$P = \left[\underbrace{700\left(1 + \frac{.06}{12}\right)^{12(5)}}_{\text{comp monthly amt in 5 yrs}}\right]\underbrace{e^{.06(15)}}_{\text{continuous 15 years}}$

$= \$2322.35$

(author says 3266.56 It would take 25.7 yrs at 6% compounded continuously to get that.)

**(7)**

$f(x) = x^{15}$

$\int x^{15}dx = \frac{x^{16}}{16} + C$

**(8)**

$\int(x^3 + 1)dx$

$= \frac{x^4}{4} + x + C$

**(9)**

$v(t) = -32t$ $\quad s_0 = 4800$

distance $s(t) = \int v(t)dt = \int -32t\,dt = \frac{-32t^2}{2} + C$

$s(t) = -16t^2 + C$

$s(0) = 4800 = -16(0)^2 + C$

$C = 4800 \quad s(t) = -16t^2 + 4800$

Hit

$0 = 16t^2 + 4800$

$t = 10\sqrt{3} \approx 17.32$ sec

Velocity

$v(17.32) = -32(17.32)$

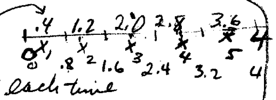
$= -554.27$ ft/sec

**(10)**

$f(x) = x^2 + x + 1 \quad 0 \le x \le 4 \quad n = 5$

$\Delta x = \frac{4-0}{5} = .8$

1st midpoint is .4 add .8 each time




$\rightarrow x_i$ left 0, .8, 1.6, 2.4, 3.2

$.8(f(0) + f(.8) + f(1.6) + f(2.4) + f(3.2))$

$= 25.76$

$\rightarrow x_i$ right

$.8(f(.8) + f(1.6) + f(2.4) + f(3.2) + f(4))$

$= 41.76$

$x_i$ midpoint

$.8(f(.4) + f(1.2) + f(2) + f(2.8) + f(3.6))$

$= 33.12$

(author says 21, 41, 30.72)

**(11)**

$f(x) = \ln x$

$1 \le x \le 5 \quad n = 2$

$\Delta x = \frac{5-1}{2} = 2$

$x_i$ left 1, 3

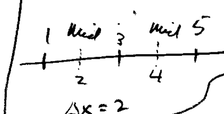
$2(\ln(1) + \ln(3)) = 2.197$

$x_i$ right 3, 5

$2(\ln 3 + \ln 5) = 5.416$

$x_i$ mid 2, 4

$2(\ln 2 + \ln 4) = 4.159$



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(12) $f(x) = \ln(x+1) \quad 0 \le x \le 1 \quad n=3$

midpts $\frac{1}{6}$, $\frac{3}{6}$, $\frac{5}{6}$ $\qquad \frac{1}{3} = \Delta x$

$0, \frac{1}{3} = \frac{2}{6}, \frac{2}{3} = \frac{4}{6}, 1$

sum $= \frac{1}{3}\left(\ln(1+\frac{1}{6}) + \ln(1+\frac{3}{6}) + \ln(1+\frac{5}{6})\right)$

$= \frac{1}{3}\left[\ln(\frac{7}{6}) + \ln(\frac{9}{6}) + \ln(\frac{11}{6})\right]$

(13) $\int_1^2 5x\,dx = \frac{5x^2}{2}\Big]_1^2 = \left[\frac{5(2)^2}{2}\right] - \left[\frac{5(1)}{2}\right]$

$= 10 - 2.5 = 7.5$ or $\frac{15}{2}$

(14) $\int_2^5 \left(e^{4x} - \frac{1}{x}\right)dx$

$= \frac{e^{4x}}{4} - \ln x \Big]_2^5$

$= \left[\frac{e^{20}}{4} - \ln 5\right] - \left[\frac{e^8}{4} - \ln 2\right]$

$= \frac{1}{4}\left[e^{20} - e^8\right] - \left[\ln 5 - \ln 2\right]$

$= \frac{1}{4}(e^{20} - e^8) + \ln 2 - \ln 5$

$= \frac{1}{4}(e^{20} - e^8) + \ln\frac{2}{5}$

(15) $f(t) = 1000e^{-.05t}$

$\int f(t)\,dt = \frac{1000}{-.05}e^{-.05t}$

$F(t) = -20000\,e^{-.05t}$

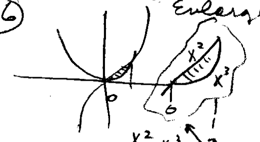
$F(2) - F(0)$

$= -20000e^{-.05(2)} - -20000$

$= -20000(e^{-.10} - 1) = 20000(1 - e^{-.1})$

$= 1903.25$

(16) Enlarge




$x^2 = x^3$

$x = 0$ or $x = 1$

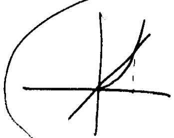
$\int_0^1 (x^2 - x^3)\,dx$

$= \frac{x^3}{3} - \frac{x^4}{4}\Big]_0^1$

$= \left(\frac{1}{3} - \frac{1}{4}\right) - (0)$

$= \frac{1}{12}$

(17) $f(x) = x^2 \quad y = x$



$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$\int_0^1 (x - x^2)\,dx$

$= \frac{x^2}{2} - \frac{x^3}{3}\Big]_0^1$

$= \left[\frac{1}{2} - \frac{1}{3}\right] - [0]$

$= \frac{1}{6}$

(18) $\int_0^2 \left(\frac{1}{2}x + 1 - e^{-x}\right)dx$

$= \frac{x^2}{4} + x + e^{-x}\Big]_0^2$

$= \left[\frac{4}{4} + 2 + e^{-2}\right] - [1]$

$= 2 + \frac{1}{e^2}$

(19) $y = Ce^{kt}$

$y = 30000e^{.04t}$

Av $= \frac{1}{6-0}\int_0^6 30000e^{.04t}\,dt$

$= \frac{1}{6-0}\cdot 30000\frac{e^{.04t}}{.04}\Big]_0^6$

$= 125000\left[e^{.04t}\right]_0^6 =$

$= 125000\left[[e^{.04(6)}] - [1]\right]$

$= 33906.14$

(20) $B = F(5) = \frac{20}{5+5} = 2$ $\qquad$ see p 352

$A = 5$

$\int_0^5 \left(\frac{20}{x+5} - 2\right)dx$

$= 20\ln(x+5) - 2x\Big]_0^5$

$= [20\ln 10 - 10] - [20\ln 5]$

$= 3.86$

(21) see p354 $\quad$ foil

$\int_0^1 \pi(x - x^3)^2\,dx$

$= \pi\int_0^1 (x^2 - 2x^4 + x^6)\,dx$

$= \pi\left[\frac{x^3}{3} - \frac{2x^5}{5} + \frac{x^7}{7}\right]_0^1$

$= \pi\left[\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right] =$

$= \frac{8\pi}{105} \approx .239$

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Also see HW Solutions for Chapter 6