ptch34_appcalc_05g10_sol.doc

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1) $f(x) = \dfrac{x^2}{x^3-5x+2}$ $\quad u = x^2 \quad u' = 2x$; $\quad v = x^3-5x+2 \quad v' = 3x^2-5$

$f'(x) = \dfrac{u'v-uv'}{v^2}$

$f'(x) = \dfrac{2x(x^3-5x+2)-x^2(3x^2-5)}{(x^3-5x+2)^2}$

$f'(x) = \dfrac{2x^4-10x^2+4x-3x^4+5x^2}{(x^3-5x+2)^2}$

$f'(x) = \dfrac{-x^4-5x^2+4x}{v^2}$

$f'(x) = \dfrac{-x(x^3+5x-4)}{(x^3-5x+2)^2}$

2) $f(x) = (x^5-8x^2+5)(x^3+\sqrt{x}-1)$

$u = x^5-8x^2+5 \quad u' = 5x^4-16x$

$v = x^3+x^{\frac{1}{2}}-1 \quad v' = 3x^2+\frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = u'v+uv'$

$f'(x) = (5x^4-16x)(x^3+x^{\frac{1}{2}}-1)+(x^5-8x^2+5)(3x^2+\frac{1}{2}x^{-\frac{1}{2}})$

$f'(x) =$ stop here or simplify to

$f'(x) = 8x^7+\frac{11}{2}x^{9/2}-45x^4+15x^2-20x^{3/2}+16x+\frac{5}{2\sqrt{x}}$

3) $f(x) = 4(x^2+1)^3(x^2+1)^4$

$u = 4(x^2+1)^3 \quad u' = 12(x^2+1)^2(2x) = 24x(x^2+1)^2 = u'$

$v = (x^2+1)^4 \quad v' = 4(x^2+1)^3(2x) = 8x(x^2+1)^3 = v'$

$f'(x) = u'v+uv'$

$f'(x) = 24x(x^2+1)^2(x^2+1)^4+4(x^2+1)^3\,8x(x^2+1)^3$

$= 24x(x^2+1)^6+32x(x^2+1)^6$

$= 56x(x^2+1)^6$

— OR — combine first

$f(x) = 4(x^2+1)^3(x^2+1)^4$

$f(x) = 4(x^2+1)^7$

$f'(x) = 28(x^2+1)^6(2x)$

$f'(x) = 56x(x^2+1)^6$

4) $f(x) = \dfrac{x^6+4x^3+1}{x^3+1}$

$u = x^6+4x^3+1 \quad u' = 6x^5+12x^2 = 6x^2(x^3+2)$

$v = x^3+1 \quad v' = 3x^2$

$f'(x) = \dfrac{u'v-uv'}{v^2}$

$f'(x) = \dfrac{6x^2(x^3+2)(x^3+1)-(x^6+4x^3+1)(3x^2)}{v^2}$

$f'(x) = \dfrac{6x^2(x^6+3x^3+2)-3x^8-12x^5-3x^2}{v^2}$

$f'(x) = \dfrac{6x^8+18x^5+12x^2-3x^8-12x^5-3x^2}{v^2}$

$f'(x) = \dfrac{3x^8+6x^5+9x^2}{v^2}$

$f'(x) = \dfrac{3x^2(x^6+2x^3+3)}{(x^3+1)^2}$

— OR —

alternate way

4) Revisited

long divide first

$$\begin{array}{r} x^3+3-\frac{2}{x^3+1} \\ x^3+1\,\overline{)\,x^6+4x^3+1} \\ \underline{x^6 \pm x^3} \\ 3x^3+1 \\ \underline{3x^3 \pm 3} \\ -2 \end{array}$$

$f(x) = x^3+3-2(x^3+1)^{-1}$

$f'(x) = 3x^2+2(x^3+1)^{-2}(3x^2)$

$f'(x) = 3x^2+\dfrac{6x^2}{(x^3+1)^2}$

$f'(x) = \dfrac{3x^2(x^3+1)^2+18x^2}{(x^3+1)^2} =$ above answer (Believe it or not)

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5. $P(x) = \dfrac{(2x-3)^4}{(6x-1)^2}$ $\begin{cases} u=(2x-3)^4 & u' = 4(2x-3)^3(2) \\ V=(6x-1)^2 & V' = 2(6x-1)(6) \end{cases}$

$P'(x) = \dfrac{u'v - uv'}{v^2}$

$P'(x) = \dfrac{8(2x-3)^3(6x-1)^2 - (2x-3)^4 \, 12(6x-1)}{v^2}$

$P'(x) = \dfrac{4(2x-3)^3(6x-1)[2(6x-1) - 3(2x-3)]}{v^2}$

$P'(x) = \dfrac{4(2x-3)^3(6x-1)(12x-2-6x+9)}{v^2}$

$P'(x) = \dfrac{4(2x-3)^3(6x-1)(6x+7)}{(6x-1)^4} = \dfrac{4(2x-3)^3(6x+7)}{(6x-1)^3}$

Set $=0$ $\quad 4(2x-3)^3(6x+7) = 0$

$2x-3=0$ — or — $6x+7=0$

possible extremes $x = \frac{3}{2}$ $\qquad x = -\frac{7}{6}$

note: $x = \frac{1}{6}$ would be a critical x-value if $P(x)$ was defined there.

6. $f(x) = \dfrac{x^2 - x}{x}$ $\quad g(x) = \dfrac{1}{\sqrt{x}}$

$f(g(x)) = \dfrac{\left(\frac{1}{\sqrt{x}}\right)^2 - \left(\frac{1}{\sqrt{x}}\right)}{\frac{1}{\sqrt{x}}} = \dfrac{\frac{1}{x} - \frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$

$= \dfrac{\frac{\sqrt{x} - x}{x^{3/2}}}{\frac{1}{x^{\frac{1}{2}}}} = \dfrac{\sqrt{x} - x}{x^{3/2}} \cdot \dfrac{x^{\frac{1}{2}}}{1}$

$= \dfrac{\sqrt{x} - x}{x}$ or $\dfrac{1}{\sqrt{x}} - 1$

8. $y = \dfrac{x}{(8-x^2)^{\frac{1}{2}}}$ $\leftarrow u$, $\leftarrow v$

$y' = \dfrac{u'v + uv'}{v^2}$

$y' = \dfrac{(1)(8-x^2)^{\frac{1}{2}} - x\left(\frac{1}{2}\right)(8-x^2)^{-\frac{1}{2}}(-2x)}{\left(\sqrt{8-x^2}\right)^2}$

$y' = \dfrac{(8-x^2)^{-\frac{1}{2}}((8-x^2) + x^2)}{8-x^2} = \dfrac{8}{(8-x^2)^{\frac{3}{2}}}$

$y'(2) = \dfrac{8}{(8-2^2)^{3/2}} = 1 = m$ $\quad$ at $(2, 5)$

$y - y_1 = m(x - x_1)$

$y - 5 = 1(x-2)$

$y = x + 3$

7. $y = u^2$ $\quad u = 3x+4$ $\quad \dfrac{du}{dx} = 3$ $\quad \dfrac{dy}{du} = 2u$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

$\dfrac{dy}{dx} = 2u \cdot 3 = 2(3x+1) \cdot 3$

$\dfrac{dy}{dx} = 6(3x+4) = 18x + 24$

— OR — $\quad y = u^2 \quad u = 3x+4$

$y = (3x+4)^2$

$\dfrac{dy}{dx} = 2(3x+4)'(3)$

$= 6(3x+4) = 18x + 24$

(9) $C = 4x + 6\sqrt{x} + 5$

$x = 2800 + 100t$

$\frac{dC}{dx} = 4 + 6\left(\frac{1}{2}\right)x^{-1/2}$

a) $\frac{dC}{dx} = 4 + \frac{3}{\sqrt{x}}$

b) substitute $x = 2800 + 100t$ in $C = 4x + 6\sqrt{x} + 5$

$C = 4(2800 + 100t) + 6\sqrt{2800 + 100t} + 5$

$C = 11200 + 400t + 6\sqrt{2800 + 100t} + 5$

$\frac{dC}{dt} = 400 + \frac{1}{2}(6)(2800 + 100t)^{-1/2}(100)$

$\frac{dC}{dt} = 400 + \frac{300}{\sqrt{2800 + 100t}}$

$\frac{dC}{dt} = \left(4 + \frac{3}{\sqrt{2800 + 100t}}\right) \cdot 100$

c) $C'(8) = \left(4 + \frac{3}{\sqrt{2800 + 100(8)}}\right) \cdot 100$

$= 405$ per week

(10)
a) $C(x) = 6x - 2\sqrt{x} + 1$

$C'(x) = 6 - 2\left(\frac{1}{2}\right)x^{-1/2}$

$C'(x) = 6 - x^{-1/2} = 6 - \frac{1}{\sqrt{x}}$

b) $C(x(t)) = 6(4t^2) - 2\sqrt{4t^2} + 1$

$C(t) = 24t^2 - 4t + 1$

$C'(t) = 48t - 4$

(11) $y = e^{-x^2/4} \rightarrow y' = e^u u'$ where $u = -\frac{x^2}{4}$

$u' = -\frac{2x}{4} = \frac{-x}{2}$

$y' = e^{\frac{-x^2}{4}}\left(-\frac{x}{2}\right)$

$y' = -\frac{x e^{\frac{-x^2}{4}}}{2}$

(12) $y = e^{3e^{2x}}$

$y' = e^u u'$

$u = 3e^{2x}$

$u' = 3e^v v'$

Imbedded

$v = 2x$

$v' = 2$

$u' = 3e^{2x} \cdot 2$

$u' = 6e^{2x}$

$y' = e^{3e^{2x}} \cdot 6e^{2x}$

$y' = 6e^{2x}e^{3e^{2x}}$

$y' = 6e^{2x + 3e^{2x}}$

(13) $f(x) = \frac{e^x + 1}{e^x - 1}$

$u = e^x + 1$

$u' = e^x$

$v = e^x - 1$

$v' = e^x$

$f'(x) = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{e^x(e^x - 1) - (e^x + 1)e^x}{(e^x - 1)^2}$

$f'(x) = \frac{e^{2x} - e^x - e^{2x} - e^x}{(e^x - 1)^2}$

$f'(x) = \frac{-2e^x}{(e^x - 1)^2}$

(14) $f(x) = e^x + e^{2x} + e^{4x}$

$= \frac{1}{e^{-4x}}$

$f'(x) = e^x + 2e^{2x} + 4e^{4x}$

(15) $f(x) = x^3 e^{-x^3}$

$u = x^3 \quad u' = 3x^2$

$v = e^{-x^3} \quad v' = -3x^2 e^{-x^3}$

$f'(x) = u'v + uv'$

$f'(x) = 3x^2 e^{-x^3} + x^3(-3x^2 e^{-x^3}) = 3x^2 e^{-x^3}(1 - x^3)$

or $f'(x) = \frac{3x^2(1 - x^3)}{e^{x^3}}$ or $3x^2 e^{-x^3} - 3x^5 e^{-x^3}$

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16) $f(t)=100+10e^{-t}-10e^{.2t}$

Find minimum value of t

$f'(t)=-10e^{-t}+2e^{.2t}=0$ ($.2(-10)=2$)

$-10e^{-t}+2e^{.2t}=0$

$-e^{-t}(10-2e^{.8t})=0$

$-e^{-t}=0$ (not possible) or $10-2e^{.8t}=0$

$2e^{.8t}=10$

$e^{.8t}=\frac{10}{2}=5$

$\ln e^{.8t}=\ln 5$

$.8t=\ln 5$

$t=\frac{\ln 5}{.8}\approx 2.01$

17) $y'=\frac{7}{6}y$

of the form $y'=ky$

so $y=Ce^{kx}$

$y=Ce^{\frac{7}{6}x}$

See p249 formula

18) $y-4y'=0$ $f(0)=\frac{2}{3}=y(0)$ ($y=2/3$, $x=0$)

$y=4y'$ of form $y'=ky$ See p249 formula

$y'=\frac{1}{4}y$ $k=\frac{1}{4}$ $y=Ce^{kx}$

$y=Ce^{\frac{1}{4}x}$ use initial value above

$y(0)=\frac{2}{3}=Ce^{\frac{1}{4}\cdot 0}$ $y=2/3$ $x=0$

$\frac{2}{3}=C\cdot 1$

$C=\frac{2}{3}$

$y=\frac{2}{3}e^{\frac{1}{4}x}$

19) $y=\frac{\ln x}{x^3}$ $u=\ln x$ $u'=\frac{1}{x}$

$y'=\frac{u'v-uv'}{v^2}$ $v=x^3$ $v'=3x^2$

$y'=\frac{\frac{1}{x}\cdot x^3-\ln x(3x^2)}{(x^3)^2}=\frac{x^2-3x^2\ln x}{x^6}$

$y'=\frac{1-3\ln x}{x^4}$

20) $y=e^x\ln 2x$ $u=e^x, u'=e^x$

$y'=u'v+uv'$ $v=\ln 2x$ ($w=2x$, $w'=2$)

$y'=e^x\ln 2x+\frac{e^x}{x}$ $v'=\frac{2}{2x}=\frac{1}{x}$

21) $y=(x+3\ln x)^4$

$y'=4(x+3\ln x)^3\left(1+\frac{3}{x}\right)$

22) $y=\frac{\ln 3x}{\ln x}$ $u=\ln 3x$ $u'=\frac{3}{3x}=\frac{1}{x}$ $v=\ln x$ $v'=\frac{1}{x}$

$y'=\frac{u'v-uv'}{v^2}=\frac{\frac{1}{x}\cdot\ln x-\ln 3x\cdot\frac{1}{x}}{(\ln x)^2}$

$y'=\frac{\ln x-\ln 3x}{x(\ln x)^2}=\frac{\ln x-\ln x-\ln 3}{x(\ln x)^2}$

$y'=\frac{-\ln 3}{x(\ln x)^2}$

23) $f(x)=\frac{\ln x}{e^x}$ $u=\ln x$ $u'=\frac{1}{x}$ $v=e^x$ $v'=e^x$

$f'(x)=\frac{u'v-uv'}{e^x}=\frac{\frac{1}{x}e^x-\ln x\cdot e^x}{e^{2x}}$

$f'(x)=\frac{e^x}{xe^{2x}}-\frac{\ln x\cdot e^x}{e^{2x}}=\frac{1}{xe^x}-\frac{\ln x}{e^x}$

24) $f(x)=x\ln(2x-x^2)$ $u=x$ $u'=1$

$f'(x)=u'v+uv'$ $v=\ln(2x-x^2)$ ← w

$f'(x)=\ln(2x-x^2)+x\frac{2-2x}{2x-x^2}$ $v'=\frac{(2-2x)}{(2x-x^2)}$ $\frac{w'}{w}$

25) $y=\ln\frac{\sqrt{x}\cdot e^x}{x^2+1}=\frac{1}{2}\ln x+x-\ln(x^2+1)$

$y'=\frac{1}{2x}+1-\frac{1}{x^2+1}\cdot 2x=\frac{1}{2x}+1-\frac{2x}{x^2+1}$

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$y = (3x+1)^5 (2x-1)^{-2} (x+3)^4$

$\ln y = 5\ln(3x+1) - 2\ln(2x-1) + 4\ln(x+3)$

$\dfrac{y'}{y} = \dfrac{5 \cdot 3}{3x+1} - \dfrac{2 \cdot 2}{2x-1} + \dfrac{4}{x+3}$

$\dfrac{y'}{y} = \dfrac{15}{3x+1} - \dfrac{4}{2x-1} + \dfrac{4}{x+3}$

$y' = (3x+1)^5 (2x-1)^{-2} (x+3)^4 \left(\dfrac{15}{3x+1} - \dfrac{4}{2x-1} + \dfrac{4}{x+3}\right)$

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$y = 4^x \cdot 5^x \cdot 6x^3$

$\ln y = x\ln 4 + x\ln 5 + \ln 6 + 3\ln x$

constants

$\dfrac{y'}{y} = \ln 4 + \ln 5 + \dfrac{3}{x}$

$\dfrac{y'}{y} = \ln 20 + \dfrac{3}{x}$

$y' = 4^x \cdot 5^x \cdot 6x^3 \left(\ln 20 + \dfrac{3}{x}\right)$