① $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x$

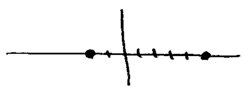
$f'(x) = x^2 - 3x - 10$

Find extremes (relative)

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$



$f''(x) = 2x - 3$

$f''(5) = 10 - 3 = 7 > 0$

concave up

Conclusion: relative min when $x = 5$

$f''(-2) = 2(-2) - 3 = -4 - 3 = -7 < 0$

concave down

Conclusion: relative max when $x = -2$

② $f(x) = x^3 - 6x$

$f'(x) = 3x^2 - 6$

$f''(x) = 6x$

when $6x$ is negative

ie) when $\frac{6x}{6} < \frac{0}{6}$

$x < 0$

Conclusion:

graph is concave down when $x < 0$.

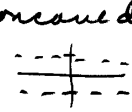
③ (1) $f' > 0$ everywhere (function is increasing throughout domain)

(2) $f''(x) > 0$ when $x < 0$ (concave up when $x < 0$)

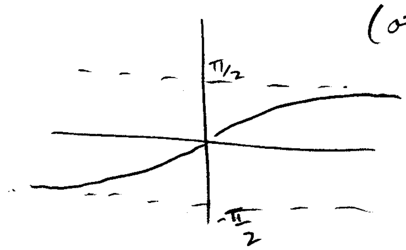
$f''(x) < 0$ when $x > 0$ (concave down when $x > 0$

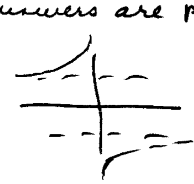
(3) asymptotes (horizontal)

when $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$



(other answers are possible for below)





④ See answer sheet

⑤ $f(x) = 2x^3 - 9x^2 + 12x - 1$

$f' = 6x^2 - 18x + 12$ ← find rel extreme

$6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 1$ or $x = 2$

use 2nd deriv to det. if max/min

$f''(x) = 12x - 18$

$f''(1) = 12 - 18 = -6 < 0$ concave down at $x = 1$

relative max is $(1, f(1)) = (1, 4)$ ← max

$f''(2) = 24 - 18 = 6 > 0$ concave up at $x = 2$

relative min is $(2, f(2)) = (2, 3)$ ← min

find inflection points

$f''(x) = 12x - 18 = 0$

$2x - 3 = 0$

$x = \frac{3}{2}$

inflection point is

$\left(\frac{3}{2}, f\left(\frac{3}{2}\right)\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

note: to verify, pick point on either side

$f''(1) = -6$ concave down

$f''(2) = 6$ concave up

ptch2_appcalc_05g10_sol.doc

(6) $f(x) = -x^3 + 3x^2 + 9x - 15$

$f'(x) = -3x^2 + 6x + 9$

$\div$ by -3 $\longrightarrow$ $x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

rel extreme $x = -1 \quad x = 3$

$f'' = -6x + 6$

$(-1, f(-1)) = (-1, -20)$ rel. extremes $f''(-1) = 12 > 0$ up

$(3, f(3)) = (3, 12)$ $f''(3) = -18 + 6 < 0$ down

inflection points

$f''(x) = -6x + 6$

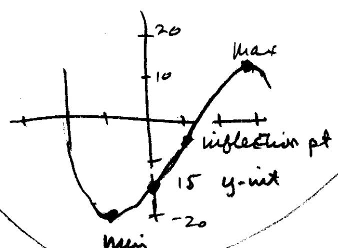
$-6x + 6 = 0$

$-x + 1 = 0$

$x = 1$

$(1, f(1)) = (1, -4)$ inflection pt.

y-int $f(0) = -15$



(8) existing Brick

$5 \qquad w \qquad w \qquad \$10$

$5 \quad l$

$5(l + w) + 10w = C$ optimize (minimize)

$5l + 15w = C$ (cost)

$lw = 300 \leftarrow$ 2nd equation Constraint

$w = \frac{300}{l}$

$C = 5l + 15\left(\frac{300}{l}\right)$

$C' = 5 - \frac{4500}{l^2}$

$-\frac{4500}{l^2} = -5$

times l^2 and divide $-5 \rightarrow$ $\frac{4500}{5} = l^2$

$l^2 = 900$

$l = \pm 30$ toss out -30

$w = \frac{300}{30} = 10$

(7) $f(x) = 9x + 1 + \frac{1}{x}, \; x > 0$

Find asymptotes

$f = 9x + 1 + \frac{1}{x}$

$f = \frac{9x^2 + x + 1}{x}$ vert at $x=0$

slant

$x \overline{)9x^2 + x + 1} = 9x + 1 = y$ slant

vert at $x = 0$

Find extremes

$f'(x) = 9 - \frac{1}{x^2}$

$9 - \frac{1}{x^2} = 0$

times x^2 $9x^2 - 1 = 0$; $9x^2 = 1$; $x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$ don't use $-1/3$ not in domain

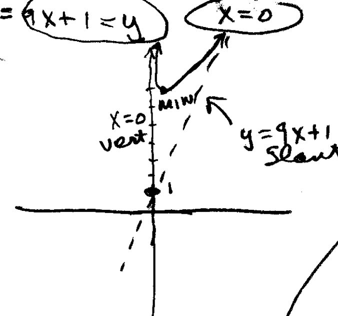
at $x = \frac{1}{3}$ relative extreme

$f'' = \frac{2}{x^3} \qquad f''\left(\frac{1}{3}\right) = \frac{2}{\left(\frac{1}{3}\right)^3} = 18 > 0$

concave up

rel min at $\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right) = \left(\frac{1}{3}, 9 \cdot \frac{1}{3} + 1 + \frac{1}{3}\right)$

$= \left(\frac{1}{3}, 7\right)$ min



(9) x = units sold

$p = 1050 - .03x \qquad C = 150x + 750000$

a) $P = R - C \qquad R = xp = x(1050 - .03x)$

$R = 1050x - .03x^2$

$P = 1050x - .03x^2 - 150x - 750000$

$P = 900x - .03x^2 - 750000$

$P' = 900 - .06x$

$900 - .06x = 0$

$x = \frac{900}{.06} = 15000$

$p(15000) = 1050 - .03(15000) = \$600$

$P(15000) = 900(15000) - .03(15000)^2 - 750000$

$= \$6{,}000{,}000$

b) cost is less by \$1,000,000
so profit is up by 1,000,000
profit is now 7,000,000

(10) $6x + 2y = 1200$ constraint

$xy = A$

$6x = 1200 - 2y$

$x = 200 - \frac{y}{3}$

$A = \left(200 - \frac{y}{3}\right)y$

$A = 200y - \frac{y^2}{3}$

$A' = 200 - \frac{2y}{3}$

$200 - \frac{2y}{3} = 0$

$\frac{2y}{3} = 200$

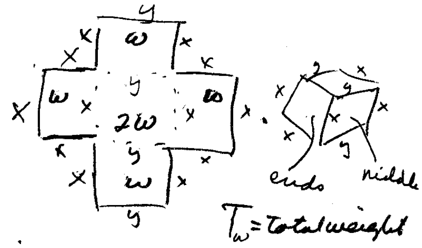
$2y = 600$

$y = 300$

$x = 200 - \frac{y}{3} = 200 - \frac{300}{3}$

$= 100 \qquad 100 \times 300$

(11)



T_w = total weight

$V = 125 = x^2y \qquad y = \frac{125}{x^2}$

$T_w = \underset{\text{ends}}{2x^2} + \underset{\text{middle}}{2xy} + \underset{\text{Bottom}}{2xy}$

$T_w = 2x^2 + 4xy$

$T_w = 2x^2 + 4x\left(\frac{125}{x^2}\right)$

$T_w = 2x^2 + \frac{500}{x}$

$T_w' = 4x - \frac{500}{x^2} = 0$

$4x = \frac{500}{x^2}$

$4x^3 = 500$

$x^3 = 125$

$x = 5$

$y = \frac{125}{5^2} = 5$

(a cube)

$C = 250n + \text{Inventory}$

$I = \frac{12500}{2n} \cdot 1$

x = # jars in inventory

n = # orders

(12) $C = 250n + \frac{12500}{2n} \cdot 1$

$C' = 250 - \frac{12500}{2n^2} = 0$

$500 = \frac{12500}{n^2}$

$n^2 = \frac{12500}{500}$

$n^2 = 25$ check

5 orders

$5\overline{)12500}$ 2500 each

10

25