ptch1_appcalc_05g10.tst Solution

① $f(x) = \frac{1}{x^2} = x^{-2}$ subtract 1

$f'(x) = -2x^{-3}$

② $y = \sqrt[3]{x^2} = x^{2/3}$

$y' = \frac{2}{3}x^{-\frac{1}{3}}$

$y'(8) = \frac{2}{3}(8)^{-1/3} = \frac{2}{3} \cdot \frac{1}{8^{1/3}}$

$y'(8) = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$m = \frac{1}{3}$, $(8, 4)$

$y - 4 = \frac{1}{3}(x - 8)$

$3y - 12 = x - 8$

$3y = x + 4$

$y = \frac{1}{3}x + \frac{4}{3}$

③ $y = \frac{x}{5} - \frac{2}{5}$

$\frac{dy}{dx} = \frac{1}{5}$

④ $\lim_{x \to -1} (x^3 - 2x + 5)$

use direct substitution

$= (-1)^3 - 2(-1) + 5$

$= 6$

⑤ use conjugate

$\lim_{x \to 0} \frac{(\sqrt{x+1} - 1)(\sqrt{x+1} + 1)}{x\ (\sqrt{x+1} + 1)}$

$= \lim_{x \to 0} \frac{x + 1 - 1}{x(\sqrt{x+1} + 1)}$

$= \lim_{x \to 0} \frac{x}{x(\sqrt{x+1} + 1)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1}$

$= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$

⑥ $\lim_{x \to -1} \frac{x^2 - 4x - 5}{x + 1}$

$= \lim_{x \to -1} \frac{(x+1)(x-5)}{x+1}$

$= \lim_{x \to -1} (x - 5)$

$= -1 - 5 = -6$

⑦ $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{(2(0+h)+1)^2 - (2 \cdot 0+1)^2}{h}$

$= \lim_{h \to 0} \frac{(2h+1)^2 - 1}{h}$

$= \lim_{h \to 0} \frac{4h^2 + 4h + 1 - 1}{h}$

$= \lim_{h \to 0} \frac{h(4h+4)}{h}$

$= \lim_{h \to 0} 4h + 4 = 4$

⑧ $\lim_{x \to \infty} \frac{x^3}{x^3 - 1}$

$= \lim_{x \to \infty} \frac{\frac{x^3}{x^3}}{\frac{x^3}{x^3} - \frac{1}{x^3}}$

$= \lim_{x \to \infty} \frac{1}{1 - \frac{1}{x^3}}$

$= \frac{1}{1 - \frac{1}{\infty}} = \frac{1}{1 - 0} = 1$

⑨ at $x = -3$ because no limit

at $x = -2$ no tangent line

at $x = 1$ no tangent line

⑩ $\lim_{x \to -3} \frac{x^3 - 9x}{2x + 6}$

$= \lim_{x \to -3} \frac{x(x-3)(x+3)}{2(x+3)}$

$= \lim_{x \to -3} \frac{x(x-3)}{2}$

$= \frac{-3(-3-3)}{2} = \frac{-3(-6)}{2}$

$= 9$

⑪ Find $P(x)$ = Demand

note: think of Demand as price.

$(\overset{x}{600}, \overset{P}{50}), (\overset{x}{800}, \overset{P}{40})$

use $P - P_1 = m(x - x_1)$

$m = \frac{50 - 40}{600 - 800} = \frac{10}{-200}$

$m = -1/20$

$P - 50 = -\frac{1}{20}(x - 600)$

$20P - 1000 = -x + 600$

$20P = -x + 1600$

$P = -\frac{x}{20} + 80$

$R = xP$

$R = x\left(-\frac{x}{20} + 80\right)$

$R = -\frac{x^2}{20} + 80x$

(12) Find limit first

$\lim_{x \to -7} \frac{x^2+2x-35}{x+7}$

$= \lim_{x \to -7} \frac{(x+7)(x-5)}{x+7}$

$= \lim_{x \to -7} (x-5)$

$= -7-5 = -12$

Fill in the hole

$f(x) = \begin{cases} \frac{x^2+2x-35}{x+7} & \text{when } x \neq -7 \\ -12 & \text{when } x = -7 \end{cases}$

NOTE: A hole in the graph is referred to as a removable discontinuity

(16) $y = \frac{4}{3x^3+x^2+4}$

$y = 4(3x^3+x^2+4)^{-1}$

$y' = -4(3x^3+x^2+4)^{-2}(9x^2+2x)$

$y' = \frac{-4(9x^2+2x)}{(3x^3+x^2+4)^2} = \frac{-4x(9x+2)}{(3x^3+x^2+4)^2}$

note: answer key is wrong

(21) $z = 4t + (3-\sqrt{2t+1})^3$

$z' = 4 + 3(3-\sqrt{2t+1})^2(-\frac{1}{2}(2t+1)^{-\frac{1}{2}}(2))$

$z' = 4 - \frac{3(3-\sqrt{2t+1})^2}{\sqrt{2t+1}}$

(22) $y = (x-4)^5$

$y' = 5(x-4)^4$

$y'' = 20(x-4)^3$

(17) $y = x^9 - 2x + (\sqrt{5-x})^3$

at $(1,7)$ i.e. when $x=1$

$y' = 9x - 2 + \frac{3}{2}(5-x)^{\frac{1}{2}}(-1)$

$y'(1) = 9(1) - 2 + \frac{3}{2}(5-1)^{\frac{1}{2}}(-1)$

$y'(1) = 9-2-3 = 4$

(23) $y = \pi + \sqrt{3}$

$y' = 0, \ y'' = 0$

(24) $f(t) = t^3 - \frac{9}{t} = t^3 - 9t^{-1}$

Find $\left.\frac{d^2f}{dt^2}\right|_{t=3}$

$f' = 3t^2 + 9t^{-2}$

$f'' = 6t - 18t^{-3}$

$f''(3) = 6(3) - \frac{18}{3^3} = 18 - \frac{18}{27}$

$f''(3) = 17\frac{1}{3} = \frac{52}{3}$

(13) $f(x) = \frac{1}{x^2+5} = (x^2+5)^{-1}$

$f'(x) = -(x^2+5)^{-2}(2x)$

$f'(x) = \frac{-2x}{(x^2+5)^2}$

(18) $y = (x^2+1)^{\frac{1}{2}}$ at $(2, \sqrt{5})$

$y' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)$

$y' = \frac{x}{\sqrt{x^2+1}}$; $y'(2) = \frac{2}{\sqrt{2^2+1}}$

$y'(2) = \frac{2}{\sqrt{5}} = m$

$y - \sqrt{5} = \frac{2}{\sqrt{5}}(x-2)$

$\sqrt{5}y - 5 = 2(x-2)$

$\sqrt{5}y = 2x - 4 + 5$

$\sqrt{5}y = 2x+1$

$y = \frac{2}{\sqrt{5}}y + \frac{1}{\sqrt{5}}$

or $y = \frac{2\sqrt{5}}{5}y + \frac{\sqrt{5}}{5}$

(14) $F(x) = \sqrt{3x+1}$

$F(x) = (3x+1)^{\frac{1}{2}}$

$F'(x) = \frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)$

$F'(x) = \frac{3}{2\sqrt{3x+1}}$ or $\frac{3}{2}(3x+1)^{-\frac{1}{2}}$

(25) $\frac{d}{dt}\left(\frac{dv}{dt}\right) = \frac{d^2v}{dt^2} = v''$

$v = -5t^3 + \frac{2}{1-t} = -5t^3 + 2(1-t)^{-1}$

$v' = -15t^2 - 2(1-t)^{-2}(-1)$

$v' = -15t^2 + 2(1-t)^{-2}$

$v'' = -30t - 4(1-t)^{-3}(-1)$

$v'' = -30t + 4(1-t)^{-3}$

or $-30t + \frac{4}{(1-t)^3}$

(15) $y = \frac{2}{\sqrt{2x+1}} = 2(2x+1)^{-\frac{1}{2}}$

$y' = 2(-\frac{1}{2})(2x+1)^{-\frac{3}{2}}(2)$

$y' = -2(2x+1)^{-\frac{3}{2}}$ or $\frac{-2}{(2x+1)^{3/2}}$

(19) $y = x^3 + 3x - 8$

~~(20)~~ at $x = 2$ note: $y(2) = 8+6-8$, $y(2) = 6$

at $(2, 6)$

$y' = 3x^2 + 3$

$y'(2) = 3(2)^2 + 3$

$y'(2) = 15$

$y - 6 = 15(x-2)$

$y = 15x - 30 + 6$

$y = 15x - 24$

(20) $y = (x^3+4x)^5$

$y' = 5(x^3+4x)^4(3x^2+4)$

26 $f(x)=3x^4-4x^3+5x+1$

$f'(x)=12x^3-12x^2+5$

$f''(x)=36x^2-24x$

$f'''(x)=72x-24$

27 $h=30t-50t^2$

$h'=v=30-100t$

$v(2)=30-100(2)$

$v(2)=-170$

28 $x=t^3-4t^2+3t$

$x'=v=3t^2-8t+3$

$x''=v'=a=6t-8$

$a(5)=6(5)-8$

$a(5)=22$

29

$s(t)=-16t^2+16t+96$

$s'(t)=v(t)=-32t+16$

$v(1)=-32(1)+16$

$v(1)=-16$

30 $s(t)=0=-16t^2+16t+96$

$0=t^2-t-6$

$0=(t-3)(t+2)$ ← toss out $t=-2$

$t=3$ seconds

31 from 29

$v(t)=-32t+16$

from 30, $t=3$

$v(3)=-32(3)+16$

$v(3)=-80$

32 at time 0, the rock is at the top of the cliff

$s(0)=-16(0)^2+16(0)+96$

$s(0)=96$ ft

33 $\dfrac{f(b)-f(a)}{b-a}$

$G(t)=t^3-t^2+.3t$

$G(0)=0$

$G(4)=64-16+1.2=49.2$

$=\dfrac{G(4)-G(0)}{4-0}$

$=\dfrac{49.2-0}{4-0}=12.3$ gal/min

34 $G'(t)=3t^2-2t+.3$

$G'(4)=3(16)-2(4)+.3$

$G'(4)=40.3$ gal/min