6.5 Selected solutions

3) $\frac{1}{b-a}\int_a^b f(x)\,dx$ = av. val of $f(x)$ see p350

$f(x)=x^3$, $a=-1$, $b=1$

$\frac{1}{1-(-1)}\int_{-1}^{1} x^3\,dx$

$= \frac{1}{2}\int_{-1}^{1} x^3\,dx$

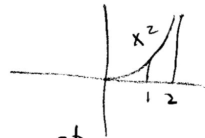
$= \frac{1}{2}\cdot\frac{x^4}{4}\Big]_{-1}^{1}$

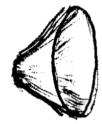
$= \frac{1}{2}\left(\left[\frac{1^4}{4}\right]-\left[\frac{(-1)^4}{4}\right]\right)$

$=0$

* I suggest also doing 1, 9, and 33 for more practice

31) $y=x^2$ $1\le x\le 2$





find volume

use see P354

$\int_a^b \pi[g(x)]^2\,dx$

$= \int_1^2 \pi(x^2)^2\,dx$

$= \int_1^2 \pi x^4\,dx$

$= \frac{\pi x^5}{5}\Big]_1^2$

$= \left[\frac{\pi(2)^5}{5}\right]-\left[\frac{\pi(1)^5}{5}\right]$

$= \frac{32\pi}{5}-\frac{\pi}{5} = \frac{31\pi}{5}$

7) $\frac{1}{12-0}\int_0^{12}\left(47+4t-\frac{1}{3}t^2\right)dt$

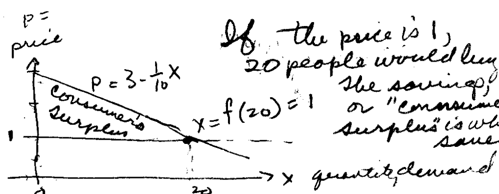
$=\frac{1}{12}\left(47t+2t^2-\frac{1}{9}t^3\Big]_0^{12}\right)$

$=\frac{1}{12}\left[47\cdot 12+2(12)^2-\frac{1}{9}(12)^3\right]-[0]$

$=\frac{1}{12}\cdot 660 = 55°$

11) Consumers Surplus See P352

$\int_0^A [f(x)-B]\,dx$



If the price is 1, 20 people would buy. The savings, or "consumer surplus" is what consumers saved by the price being lowered by demand. (because some would have bought at a higher price)

A = quantity demanded (sales level)

demand (price) = $p=f(x)$

$B=f(A)$

$f(x)=p=3-\frac{x}{10}$; $x=20$ $B=3-\frac{20}{10}=1$

$A=20$ $\int_0^{20}\left[3-\frac{x}{10}-1\right]dx = 3x-\frac{x^2}{20}-x\Big]_0^{20}$

$= 2x-\frac{x^2}{20}\Big]_0^{20} = \left[2(20)-\frac{20^2}{20}\right]-[0]$

$40-20$

$=20$

6.5

9

$1 = 2e^{k(1690)}$ NOTE $\ln\frac{1}{2} = -\ln 2$

$\frac{-\ln 2}{1690} = k$ initial amount

$y = 100e^{\frac{-\ln 2}{1690}t}$

$\frac{1}{1000-0}\int_0^{1000} 100e^{\frac{-\ln 2}{1690}t}\,dt$

$= \frac{1}{1000}\left[\frac{100e^{\frac{-\ln 2}{1690}t}}{\frac{-\ln 2}{1690}}\right]_0^{1000}$

$= \frac{1}{1000}\left[\frac{169000}{-\ln 2}e^{\frac{-\ln 2}{1690}t}\right]_0^{1000}$

$= \left[\frac{169}{-\ln 2}\,e^{\frac{-\ln 2}{1690}\cdot 1000}\right] - \left[\frac{169}{-\ln 2}\right]$ $e^0=1$

$= 82.03$ grams

33

$\int_0^4 \pi(\sqrt{x})^2\,dx$

$= \int_0^4 \pi x\,dx$

$= \left.\frac{\pi x^2}{2}\right]_0^4$

$= \frac{16\pi}{2} = 8\pi$

1 $x^2 \quad 0 \le x \le 3$

$\frac{1}{3-0}\int_0^3 x^2\,dx$

$= \frac{1}{3}\left.\frac{x^3}{3}\right]_0^3 = \frac{1}{3}\left(\left[\frac{3^3}{3}\right] - [0]\right)$

$= \frac{27}{9} = 3$