PQ2

(22) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$f_x = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}(2x)$

$f_x(3,\sqrt{11},-4) = \frac{1}{6}(3) = \frac{1}{2}$

note: author says (c) but I get (a)

However, $f_z(3,\sqrt{11},-4) = -\frac{2}{3}$

(23) $xy^2 + xz^2 - 10 = 0 = F(x,y,z)$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{y^2+z^2}{2zx}$$

(24) $\frac{dr}{dt} = -4 \quad \frac{dh}{dt} = 8 \quad r=4,\ h=8$

find $\frac{dV}{dt}$ $\quad V = \pi r^2 h$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$\frac{dV}{dt} = 2r\pi h\frac{dr}{dt} + \pi r^2\frac{dh}{dt}$$

$$\frac{dV}{dt} = 2(4)\pi(8)(-4) + \pi(4)^2(8)$$

$$\frac{dV}{dt} = -128\pi$$

(25) $f(x,y) = 3x^2 - 3xy - y^2$

$\max[D_u f(1,1)] = \|\nabla f\|$

$\nabla f = \langle 6x-3y, -3x-2y\rangle$

$\nabla f(1,1) = \langle 3, -5\rangle$

$\|\nabla f(1,1)\| = \sqrt{9+25} = \sqrt{34}$

(26) $x+2y+3z = 6 \quad P(3,0,1)$

$F(x,y,z) = x+2y+3z-6$

$\nabla F = \langle 1,2,3\rangle$

$$\frac{\nabla F}{\|\nabla F\|} = \frac{\langle 1,2,3\rangle}{\sqrt{14}}$$

(27) $2x^2+3y^2+4z^2 = 18 \quad P(-1,2,1)$

$F(x,y,z) = 2x^2+3y^2+4z^2-18$

$\nabla F = \langle 4x, 6y, 8z\rangle \quad \nabla F(-1,2,1) = \langle -4, 12, 8\rangle = n_1$

$n \cdot \overrightarrow{PQ} = 0 = \langle 1,-3,-2\rangle \cdot \langle x+1, y-2, z-1\rangle$

$0 = x+1-3y+6-2z+2$

$0 = x-3y-2z+9$

or $x-3y-2z = -9$

(28) $x^2+y^2-z^2 = 0 \quad P(-3,4,5)$

$F(x,y,z) = x^2+y^2-z^2$

$\nabla F = \langle 2x, 2y, -2z\rangle \quad \nabla F(-3,4,5) = \langle -6, 8, -10\rangle$

Let $\langle a,b,c\rangle = \langle 3,-4,5\rangle$

$\overrightarrow{PQ} = t\langle a,b,c\rangle$

$\langle x+3, y-4, z-5\rangle = t\langle 3,-4,5\rangle$

$$\frac{x+3}{3} = \frac{y-4}{-4} = \frac{z-5}{5}$$

or $$\frac{x+3}{-6} = \frac{y-4}{8} = \frac{z-5}{-10}$$

(29) $\overset{z}{f(x,y)} = x^2-3xy+y^2 \quad P(1,1)$

$F(x,y,z) = x^2-3xy+y^2-z$

$\nabla F = \langle 2x-3y, -3x+2y, -1\rangle$

$\nabla F(1,1) = \langle -1,-1,-1\rangle = v_1 \quad v = \langle 1,1,1\rangle$

$$u = \frac{v}{\|v\|} = \frac{\langle 1,1,1\rangle}{\sqrt{3}}$$

(30) $d = 2\cdot 8 - 16 = 0$

Test is inconclusive