PQ2

(Note: * In addition to the PQ$_1$ test questions may also be similar to the ones on the daily group worksheets)

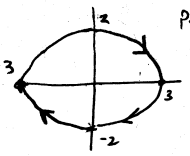
① $r(t) = 3t\,i - \sqrt{1-t^2}\,j + 4k$

$\mathbb{R}$; $1-t^2 \geq 0$, $t^2 \leq 1$; $\mathbb{R}$

$\mathbb{R} \cap \{t \mid -1 \leq t \leq 1\} \cap \mathbb{R} = \{t \mid -1 \leq t \leq 1\}$

② $r(t) = 3\cos(2t)\,i - 2\sin(2t)\,j$

t	x	y
0	3	0
$\pi/4$	0	-2
$\pi/2$	-3	0
$3\pi/4$	0	2
π	3	0



Period π

③ $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Let $x = 5\cos t$

Then $\frac{(5\cos t)^2}{25} + \frac{y^2}{16} = 1$

$\cos^2 t + \frac{y^2}{16} = 1$

↑ have to be $\sin^2 t$ (since $\cos^2 t + \sin^2 t = 1$)

$\frac{y^2}{16} = \sin^2 t$

$y = 4\sin t$

$r(t) = x\,i + y\,j$

$r(t) = 5\cos t\,i + 4\sin t\,j$

④ $x = y^2 - 1$

If $x = t$ then

$t = y^2 - 1$

and $y = \sqrt{t+1}$.

∴ (a) and (b) no good.

If $x = t^2$ then

$t^2 = y^2 - 1$ and $y = y = \sqrt{t^2+1}$

∴ (c) is no good.

If $y = t$, then $x = t^2 - 1$.

∴ (d) is no good ans (e)

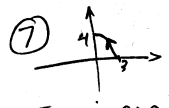
⑤ $\lim_{t \to 0} \left[t\,i + \frac{\sin t}{t}\,j + \frac{1}{e^t}\,k \right]$

$= 0\,i + j + k = j + k$

⑥ $r(t) = \frac{1}{t}\,i + \ln(-t)\,j + t^2 k$

↑ $t \neq 0 \cap t < 0 \cap \mathbb{R} = t < 0$

⑦



enter in PAR mode $X_1T = \sqrt{8 - T^3}$, $Y_1T = T^2$

⑧ $r'(t) = \frac{1}{1+t^2}\,i + e^{-t}\,k$

$r(0) = i - k$

$r = \langle \arctan t + C_1, C_2, -e^{-t} + C_3 \rangle$

$r(0) = \langle 1, 0, -1 \rangle$

$\arctan 0 + C_1 = 1$, $C_2 = 0$, $-e^{-0} + C_3 = -1$

$C_1 = 1$, $C_2 = 0$, $C_3 = 0$

$r = \langle \arctan t + 1, 0, -e^{-t} \rangle$

⑨ $\int_0^{\pi} (\sin t\,i + t^2 j + k)\,dt$

$= -\cos t\,i + \frac{t^3}{3}\,j + t\,k \Big]_0^{\pi}$

$= (1+1)i + \frac{\pi}{3}^3 j + \pi k$

$= 2i + \frac{\pi}{3}^3 j + \pi k$

(no answer (e))

⑩ $\int_0^1 \left(e^{-t}\,i + \frac{1}{t+1}\,j \right.$

$\langle -e^{-t}, \ln(t+1) \rangle \Big]_0^1$

$\langle -\frac{1}{e} + 1, \ln 2 \rangle$

⑪ $a(t) = \langle 0, e^t, -32 \rangle$

$v(0) = \langle 3, -2, 1 \rangle$

$v(t) = \langle C_1, e^t + C_2, -32t + C_3 \rangle$

$v(0) = \langle 3, -2, 1 \rangle = \langle C_1, 1 + C_2, C_3 \rangle$

$C_1 = 3 \quad C_2 = -3 \quad C_3 = 1$

$v(t) = \langle 3, e^t - 3, -32t + 1 \rangle$

⑫ $T(t) = \frac{1}{\sqrt{9t^2+5}} \langle 3t, 1, -2 \rangle$

$T' = \frac{1}{9t^2+5} \langle \quad \rangle$

$\langle 3(9t^2+5)^{\frac{1}{2}-\frac{1}{2}}(3t)(9t^2+5)^{-\frac{1}{2}}(18t),$

$-\frac{1}{2}(9t^2+5)^{-\frac{1}{2}}(18t),$

$2\left(\frac{1}{2}\right)(9t^2+5)^{-\frac{1}{2}}(18t) \rangle$

$= \langle 3(9t^2+5)^{-\frac{1}{2}} - 27t^2(9t^2+5)^{-\frac{3}{2}},$

$-9t(9t^2+5)^{-\frac{3}{2}}, 18t(9t^2+5)^{-\frac{3}{2}} \rangle$

$= \frac{1}{(9t^2+5)^{\frac{3}{2}}} \langle 15, -9t, 18t \rangle$

$= \frac{3}{(9t^2+5)^{\frac{3}{2}}} \langle 5, -3t, 6t \rangle$