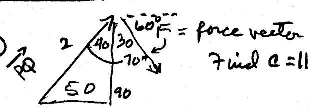
PQ1 Solutions

(11) 

$M = \overrightarrow{PQ} \times F$

$\overrightarrow{PQ} = 2\langle \cos 50°, \sin 50° \rangle$

$F = c\langle \cos -60°, \sin -60° \rangle$

$$200 = \overrightarrow{PQ} \times F = \begin{vmatrix} i & j & k \\ 2\cos 50° & 2\sin 50° & 0 \\ c\cos -60° & c\sin -60° & 0 \end{vmatrix}$$

$$200 = 2ck\underbrace{(\cos 50° \sin(-60°) - \sin 50° \cos(-60°))}_{m}$$

note: k is the only component

$\|200\| = 2c(\cos 50° \sin(-60°) - \sin 50° \cos(-60°))$

$\|2ckm\| = |2cm| \|k\| = |2cm|$ ($\|k\| = 1$)

$100 = |c(-.939692621)|$

$c = 106.42$ lbs.

(12) Thru $(1, 2, 3)$ [P], $\parallel$ to $x = y = z$

$x = t \quad y = t \quad z = t \qquad Q(x, y, z)$

direction vector $\Rightarrow \langle a, b, c \rangle = \langle 1, 1, 1 \rangle$

$\overrightarrow{PQ} = t\langle a, b, c \rangle$

$\langle x-1, y-2, z-3 \rangle = \langle t, t, t \rangle$

$x - 1 = t, \quad y - 2 = t, \quad z - 3 = t$

par $x = 1 + t, \quad y = 2 + t, \quad z = 3 + t$

sym $x - 1 = y - 2 = z - 3$

(13) Thru $(0, 1, 4)$ [P] $\perp$ to $\langle 2, -5, 1 \rangle = u$, $\langle -3, 1, 4 \rangle = v$

$Q(x, y, z)$

direction vector CROSSP

$u \times v = \langle -21, -11, -13 \rangle = \langle a, b, c \rangle$

$\overrightarrow{PQ} = t\langle a, b, c \rangle$

$\langle x, y-1, z-4 \rangle = \left[\begin{array}{l} t\langle -21, -11, -13 \rangle \\ \text{use } t\langle 21, 11, 13 \rangle \end{array}\right]$

par $x = 21t, \quad y = 1 + 11t, \quad z = 4 + 13t$

sym $\dfrac{x}{21} = \dfrac{y-1}{11} = \dfrac{z-4}{13}$

↑ Reminder: not unique

(14) plane thru $(-3, -4, 2)$ [P], $(-3, 4, 1)$ [R] & $(1, 1, -2)$ [S]; $Q(x, y, z)$

$u = \overrightarrow{PR} = \langle 0, 8, -1 \rangle$

$v = \overrightarrow{PS} = \langle 4, 5, -4 \rangle$

$n = u \times v = \langle -27, -4, -32 \rangle$

$\overrightarrow{PQ} = \langle x+3, y+4, z-2 \rangle$

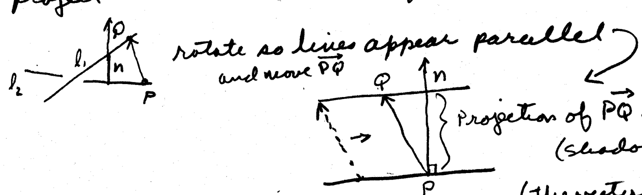
$n \cdot \overrightarrow{PQ} = 0$ use $\overrightarrow{PQ}$

$\langle -27, -4, -32 \rangle \cdot \langle x+3, y+4, z-2 \rangle = 0$

$-27(x+3) - 4(y+4) - 32(z-2) = 0$

$27x - 4y - 32z - 33 = 0$

(15) note: The lines intersect so D = 0. This is to show a method only. The cross product of the direction vectors will be $\perp$ to both lines [n]. Then, after finding a point on each line, we project the vector formed by them onto n to get w_1. $D = \|w_1\|$



l_1: $\dfrac{x-4}{1} = \dfrac{y-3}{-1} = \dfrac{z-7}{3}$ and l_2: $\dfrac{x+3}{5} = \dfrac{y-7}{2} = \dfrac{z+5}{-6}$

Find n, $\langle a, b, c \rangle = \langle 1, -1, 3 \rangle \qquad \langle a, b, c \rangle = \langle 5, 2, -6 \rangle$

$n_1 = \langle 1, -1, 3 \rangle \times \langle 5, 2, -6 \rangle = \langle 0, 21, 7 \rangle$

use $n = \langle 0, 3, 1 \rangle$

Find Points l_1: $x - 4 = t, \; y - 3 = -t, \; z - 7 = 3t$

let $t = 0$

$x = 4, y = 3, z = 7$

$Q(4, 3, 7)$

l_2: $x + 3 = 5s \quad y - 7 = 2s \quad z + 5 = -6s$

let $s = 0$

$x = -3 \quad y = 7 \quad z = -5$

$P(-3, 7, -5)$

$\overrightarrow{PQ} = \langle -7, 4, -12 \rangle$

$$D = \|\text{proj}_n \overrightarrow{PQ}\| = \left\| \frac{\overrightarrow{PQ} \cdot n}{\|n\|^2} n \right\| = |\overrightarrow{PQ} \cdot n| \frac{\|n\|}{\|n\|^2} = \frac{|\overrightarrow{PQ} \cdot n|}{\|n\|}$$

$$= \frac{|\langle -7, 4, -12 \rangle \cdot \langle 0, 3, 1 \rangle|}{\|\langle 0, 3, 1 \rangle\|} = \frac{0 + 12 - 12}{\sqrt{10}} = 0 = D$$

the lines intersect because D = 0 but they are not the same lines