PQ1 Solution

(5) $u = \langle -4, 3, -6 \rangle$, $v = \langle 16, -12, 24 \rangle$

since $v = -4u$, u and v are parallel.

(6) $u = 5\left[\cos \frac{3\pi}{4}\, i + \sin \frac{3\pi}{4}\, j\right]$

$v = 2\left[\cos \frac{2\pi}{3}\, i + \sin \frac{2\pi}{3}\, j\right]$

$u = 5\left(-\frac{\sqrt{2}}{2}\right) i + 5\left(\frac{\sqrt{2}}{2}\right) j$

$u = \left\langle \frac{-5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$

$v = 2\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

$v = \langle -1, \sqrt{3} \rangle$

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|} = \frac{\frac{5\sqrt{2}}{2} + \frac{5\sqrt{6}}{2}}{\sqrt{\frac{25\cdot 2}{4} + \frac{25\cdot 2}{4}}\sqrt{1+3}}$$

$$\cos\theta = \frac{5(\sqrt{2}+\sqrt{6})}{2\cdot 5(2)} = \frac{\sqrt{2}+\sqrt{6}}{4}; \quad \theta = 15^\circ$$

(7) Component form of u when $u \perp$ to $x - 3y + 4z = 0$ and $\|u\| = 3$

$n = \langle 1, -3, 4 \rangle$ (n is $\perp$ to plane)

unit vector for n is

$$\frac{n}{\|n\|} = \frac{\langle 1, -3, 4 \rangle}{\sqrt{1+9+16}} = \left\langle \frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle.$$

Then $u = 3 \cdot \frac{n}{\|n\|} = \left\langle \frac{3}{\sqrt{26}}, \frac{-9}{\sqrt{26}}, \frac{12}{\sqrt{26}} \right\rangle$

$$u = \frac{3}{\sqrt{26}} i - \frac{9}{\sqrt{26}} j + \frac{12}{\sqrt{26}} k$$

(8) Component form of u when u is $\perp$ to

(1) $x = 4 - t$, $y = 3 + 2t$, $z = 1 + 5t$

(2) $x = -3 + 7s$, $y = -2 + s$, $z = 1 + 2s$ direction vectors

For (1), $\langle a, b, c \rangle = \langle -1, 2, 5 \rangle = v$

For (2), $\langle a, b, c \rangle = \langle 7, 1, 2 \rangle = w$

$v \times w = \langle -1, 2, 5 \rangle \times \langle 7, 1, 2 \rangle = \langle -1, 37, -15 \rangle$

$u_1 = -i + 37j - 15k$ $\quad u = \frac{u_1}{\|u_1\|} = \frac{\langle -1, 37, -15 \rangle}{\|\langle -1, 37, -15 \rangle\|} = \frac{1}{\sqrt{1595}} \langle -1, 37, -15 \rangle$

(9) $u = \langle 3, -2, 1 \rangle$, $v = \langle 2, -4, -3 \rangle$ and $w = \langle -1, 2, 2 \rangle$

a) Find $\|u\| = \sqrt{9+4+1} = \sqrt{14}$

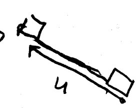
b) $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|} = \frac{6 + 8 - 3}{\sqrt{14}\sqrt{4+16+9}} = \frac{11}{\sqrt{14}\sqrt{29}}$; $\theta = 56.91^\circ$

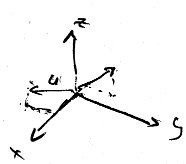
c) $\frac{v \times w}{\|v \times w\|} = \frac{\langle -2, -1, 0 \rangle}{\sqrt{4+1+0}} = -\frac{2}{\sqrt{5}} i - \frac{1}{\sqrt{5}} j$

d) $\text{proj}_u w = \frac{w \cdot u}{\|u\|^2} u = \frac{-3 - 4 + 2}{9+4+1} \langle 3, -2, 1 \rangle$

$= \frac{-5}{14} \langle 3, -2, 1 \rangle = -\frac{15}{14} i + \frac{5}{7} j - \frac{5}{14} k$

e) $\xrightarrow{w}$





use answer from d.

$W = \|\text{proj}_u w\| \|u\|$

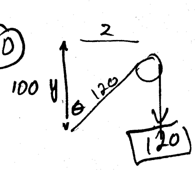
$= \left\| \left\langle -\frac{15}{14}, \frac{5}{7}, -\frac{5}{14} \right\rangle \right\| \|\langle 3, -2, 1 \rangle\|$

$= \frac{1}{14}\sqrt{225 + 100 + 25}\,\sqrt{9+4+1}$

$= \frac{1}{14}\sqrt{350}\,\sqrt{14} = 5$

or use $W = |w \cdot u| = |-3 - 4 + 2| = |-7+2| = 5$ (get mag.)

(10)



$\tan\theta = \frac{2}{y}$

$\cos\theta = \frac{100}{120} = \frac{5}{6}$

$\theta = \arccos \frac{5}{6}$

$y = \frac{2}{\tan(\arccos \frac{5}{6})} = 3.02$ ft

See the last page for an alternate way to do #10 using vectors.