

Practice Quiz 1 Solutions

① $P(-2, -1), Q(5, -1), R(2, 4)$

$u = \vec{PQ} = \langle 7, 0 \rangle$

$v = \vec{PR} = \langle 4, 5 \rangle$

a) $u = 7i$

$v = 4i + 5j$

b) $\|v\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

c) $u \cdot v = \langle 7, 0 \rangle \cdot \langle 4, 5 \rangle$

$u \cdot v = 28 + 0 = 28$

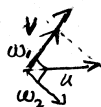
d) $2u + v =$

$2(7i) + (4i + 5j) =$

$14i + 4i + 5j =$

$18i + 5j$

e) $w_1 = \text{proj}_v u$



$w_1 = \frac{u \cdot v}{\|v\|^2} v$

$= \frac{\langle 7, 0 \rangle \cdot \langle 4, 5 \rangle}{\| \langle 4, 5 \rangle \|^2} \langle 4, 5 \rangle$

$= \frac{28}{41} \langle 4, 5 \rangle = \frac{112}{41} i + \frac{140}{41} j$

f)

$w_2 = u - w_1 = \langle 7, 0 \rangle - \langle \frac{112}{41}, \frac{140}{41} \rangle$
 $= \langle \frac{175}{41}, -\frac{140}{41} \rangle$

② Endpoints $(0, 0, 4), (4, 6, 0)$

$r = \frac{1}{2} \sqrt{(0-4)^2 + (0-6)^2 + (4-0)^2}$

$r = \frac{1}{2} \cdot 2\sqrt{17} = \sqrt{17}$

$c = \left(\frac{0+4}{2}, \frac{0+6}{2}, \frac{4+0}{2} \right) = (2, 3, 2)$

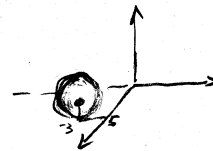
$(x-2)^2 + (y-3)^2 + (z-2)^2 = \sqrt{17}^2$

③ $x^2 + y^2 + z^2 - 10x + 6y - 4z + 34 = 0$

$x^2 - 10x + 25 + y^2 + 6y + 9 + z^2 - 4z + 4 = -34 + 25 + 9 + 4$

$(x-5)^2 + (y+3)^2 + (z-2)^2 = 4$

center: $(5, -3, 2)$ $r = 2$



④ $P(2, -1, 3), Q(0, 5, 1)$

and $R(5, 5, 0)$

$u = \vec{PQ} = \langle -2, 6, -2 \rangle$

$v = \vec{PR} = \langle 3, 6, -3 \rangle$

a) $u = -2i + 6j - 2k$

$v = 3i + 6j - 3k$

b) $u \cdot v = \langle -2, 6, -2 \rangle \cdot \langle 3, 6, -3 \rangle$

$u \cdot v = -6 + 36 - 6 = 36$

c) $u \times v = \begin{vmatrix} i & j & k \\ -2 & 6 & -2 \\ 3 & 6 & -3 \end{vmatrix} = (-18 + 12)i - (6 + 6)j + (-12 - 18)k$

$u \times v = -6i - 12j - 30k$

d) Equation of Plane thru $P, Q,$ and R

$n_1 = u \times v = \langle -6, -12, -30 \rangle$ use $\langle 1, 2, 5 \rangle = n$

$\vec{PQ} = \langle x-2, y+1, z-3 \rangle$

$\vec{PQ} \cdot n = x-2 + 2y+2 + 5z-15 = 0$

$x + 2y + 5z = 15$

not unique

e) direction vector $\langle a, b, c \rangle = \vec{PQ} = \langle -2, 6, -2 \rangle$

$\langle x-2, y+1, z-3 \rangle = t \langle -2, 6, -2 \rangle$

$x = 2 - 2t$ $y = -1 + 6t$ $z = 3 - 2t$

$\langle x-0, y-5, z-1 \rangle = t \langle -2, 6, -2 \rangle$

$x = -2t$ $y = 5 + 6t$ $z = 1 - 2t$