difference_quotients evolution to 09/17/02 5:43 PM (last update)

Calculating a Difference Quotient

General Information: Difference quotients are used in calculus as part of the process used to determine the instantaneous rate of change (derivative) of a function (or relation). For students who do not plan to take the calculus, they are used as an exercise in the algebra of functions or to calculate "average rates of change".

The numerator is the difference of the same function using two different function arguments, namely, *x* and *x* + *h*.

Difference quotient: $\frac{f(x+h)-f(x)}{h}$

$f(x)$
$f(x+h)$ Note: $f(x+\Delta x)$ is sometimes used in place of $f(x+h)$.

Process:
f, g, h, etc. are commonly used function names while *x, y, z, etc.* are commonly used independent variables. This allows us to refer to a function by name such as *f* (*x*), *g*(*y*), *u*(*k*), etc. This names are especially useful when we have more than one function at hand.

For example, $f(x)=x^3-x+1$ defines a function called *f* with the independent variable *x*. The definition part, x^3-x+1, tells us what is done with the independent variable *x* if we wanted to put in a value for it. For example, $f(2)=2^3-2+1=7$.

In the function $f(x)=x^3-x+1$, the *x* acts as a place holder. If we want, for example, to find "*f* of 3", we replace every occurrence of *x* by a 3 to get $f(3)=(3)^3-3+1=27-3+1=25$. In the example that follows, we want to find "*f* of *x* + *h*" and "*f* of *x*" as part of the process of finding the difference quotient. So we will replace all *x*'s in $f(x)=x^3-x+1$ by *x* + *h* in the $f(x+h)$ part of the numerator, and just use $f(x)$, as is, in the second part.

One way to calculate a difference quotient is to first set up $f(x)$ and $f(x+h)$. Then insert them in the difference quotient. We then simplify the expression by using algebra.

Difference quotient: $\frac{f(x+h)-f(x)}{h}$

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Example) Find the difference quotient for $f(x) = x^3 - x + 1$

First, set up $f(x)$ and $f(x+h)$.

$f(x) = x^3 - x + 1$ (no calculation required…just copy the function)

$f(x+h) = (x+h)^3 - (x+h) + 1$ (replace all occurrences of x in $f(x) = x^3 - x + 1$ by $x + h$)

Then insert them in the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^3 - (x+h) + 1] - [x^3 - x + 1]}{h}$$

Next, simplify the expression by using algebra. Our goal is to have the h factor out and cancel. If that doesn't happen, find your mistake!

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^3 - (x+h) + 1] - [x^3 - x + 1]}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x^3 + 3x^2h + 3xh^2 + h^3) - (x+h) + 1] - [x^3 - x + 1]}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x - h + 1 - x^3 + x - 1}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3 - h}{h} = \frac{h(3x^2 + 3xh + h^2 - 1)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2 - 1$$

Note: In calculus, we would let the h in $3x^2 + 3xh + h^2 - 1$ go to zero and we would have $3x^2 + 3x \cdot 0 + 0^2 - 1 = 3x^2 - 1$ as the "derivative".